JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTERS OF SCIENCE IN PURE AND APPLIED MATHEMATICS

COURSE CODE: SMA 801

COURSE TITLE: ABSTRACT INTEGRATION I

DATE: 1/03/2013 **TIME: 2.00-5.00PM**

<u>INSTRUCTIONS</u>

1. This examination paper contains **FIVE** questions.
2. Answer **any THREE questions**.

QUESTION ONE (20 marks)

a) Show that every Borel subset of P is Lebesgue-measurable (6 marks)

b) Show that if E_n is a Lebesque-measurable subset of P for each $n \in$ N, then U{ $E_n : n \in$ N } is also Lebesque-measurable (10 marks)

c) Using cardinality arguments and the cantor set P of P show that the Borel algebra B(P) is a proper subset of M (the class of all lebesgue-measurable subsets of P)
(Quote results you use without proofs) (4 marks)

QUESTION TWO (20 marks)

a) Let (X,S) be a measurable space and the function $f,g : X \to \mathbf{R}^*$ be S-measurable. Quoting all the results you use (without proving them) give an outline of the solution to prove that the product $f.g$ is S-measurable. (6 marks)

b) Let (X,S) be a measurable space and $f : X \to C$ be a function. Show that $f^{-1}(x) \in S$ for each Borel set B of X if and only if the real-valued functions $f_1 = \text{Re } f$ and $f_2 = \text{Im } f$ are S-measurable. (8 marks)

c) Let (X,S) be a measurable space and $f : X \to \mathbf{R}^*$ is a S-measurable non-negative function. Show that there is a monotonic increasing (S) of a non-negative S-measurable real-valued simple functions converging to f on X (6 marks)

QUESTION THREE (20 marks)

a) State and prove Monotone Convergence theorem (13 marks)

b) Let (X,S_μ) be a measure space and f be S-measurable on X and non-negative. Show that $\int fd\mu \geq 0$ with equality holding if and only if $f = 0$ $\mu - ae.$ (7 marks)

QUESTION FOUR (20 marks)

a) (X,S,μ) is an incomplete measure space. Show that we can always obtain a complete measure space (X,S_0,μ_0) where $S_0 \supset S$ and $\mu_0(E) = \mu(E)$ for all $E \in S$. (14 marks)

b) Let (X,S,μ) be an incomplete measure space and f,g be functions on X into $\mathbf{R}^*$ such that $f = g$ $\mu - ae.$ Let f be S-measurable and let $g(x) =$ a constant c for all $x \in E = \{x \in X : f(x) \neq g(x)\}.$ Show that g is S-measurable. (8 marks)

QUESTION FIVE (20 marks)

a) Let (X,S,μ) be a measure space and f,g be extended real value functions on X such that $f = g$ $\mu - ae$ and such that f and g are both S-measurable. If f is integrable show that g is integrable and that $\int Fd\mu = \int gd\mu$. (10 marks)

b) State and prove the Dominated convergence theorem for a sequence of complex-valued measurable functions. (10 marks)