JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTERS OF SCIENCE IN PURE AND APPLIED MATHEMATICS

COURSE CODE: SMA 805

COURSE TITLE: GENERAL TOPOLOGY I

DATE: 2/03/2013 **TIME: 9.00 AM -12.00 PM**

INSTRUCTIONS

1. This examination paper contains **FIVE** questions.
2. Answer **any THREE questions**.

QUESTION ONE (20 marks)

a) Let (x, y) be a topological space. Show that E is nowhere dense if and only if $X - E$ is dense in X. (Derive any result that you may use in your solution) (7 marks)

b) Let (x, y) be a topological space. If A is the intersection of all the closed subsets of X that contain a subset E of X, show that every point of A is a point of adherence of E. (8 marks)

c) If Λ is an index set of infinite cardinality and $\{E_\alpha : \alpha \in \Lambda\}$ is a family of subsets of a topological space (x, y), show that

$$\overline{\underset{\alpha\in\Lambda}{U} E_\alpha} \supset \underset{\alpha\in\Lambda}{U} \overline{E}_\alpha \qquad (*)$$

But that the set equality need not in $(*)$ (5 marks)

QUESTION TWO (20 marks)

a) Quote a set of necessary and sufficient conditions for a family of subsets of a nonvoid set X to be a base for a topology in X and prove your statement. (8 marks)

b) Show that any nonvoid collection Ϭ of subsets of a nonvoid set X can serve as a subbasis for a topology in X and show that this topology is the intersection of all the topologies on X that contain Ϭ. (7 marks)

c) Consider $\mathbf{R}$ and let $B_1 = \{(a,b) : a,b \in \mathbf{R} \text{ and } a < b\}$, $B_2 = \{[a,b) : a,b \in \mathbf{R} \text{ and } a < b\}$ be a family of all subintervals of $\mathbf{R}$ as indicated. If y_1 and y_2 are the topologies generated by B_1, B_2 respectively, show that y_2 is strictly finer than y_1 (5 marks)

QUESTION THREE (20 marks)

a) Let X be a nonvoid set and $k : P(x) \to P(x)$ an operator which satisfies the following properties:

(i) $k(\phi) = \phi$ (ii) $k(E) \supseteq E$ for all $E \in P(X)$. (iii) $k(A \cup B) = k(A) \cup k(B)$ for all $A, B \in P(X)$ (iv) $k(k(E)) = k(E)$ for all $E \in P(X)$

Let $\Im$ be the family $\{F \in P(X) : k(F) = F\}$ and $y = \{F^c : F \in \Im\}$. Show that y is a topology on X and that if $\overline{E}$ is the closure of E with respect to this topology, then $k(E) = \overline{E}$ for all $E = P(X)$ (10 marks)

b) (x, y), (Y, u) are topological spaces. Show that the following statements are equivalent:

i. f^{-1} is closed with respect to (x, y) for each subset F of Y which is closed with respect to u.

ii. $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \in P(x)$

iii. $\overline{f^{-1}(B)} = f^{-1}(\overline{B})$ for all $B \in P(y)$ (10 marks)

QUESTION FOUR (20 marks)

a) Show by means of an example that if a function is sequentially continuous at a point in a topological space, it need not to be continuous there at (8 marks)

b) If (x, y) is a first countable topological space and (Y, u) is any topological space and $f : X \to Y$ is a function, show that sequential continuity of f at a point $p = X$ does imply that f is continuous at p with respect to y and u. (7 marks)

c) Show that the cofinite topology on the real line $\mathbf{R}$ is not first countable (5 marks)

QUESTION FIVE (20 marks)

a) State Urysohn's lemma for a normal space. (2 marks)

b) If (x, y) is a second countable space, prove

   i. every open cover of any subset A of X is reducible to a countable cover (7 marks)

   ii. every base Y for y is reducible to a countable base.(6 marks)

c) Show that a topological space (x, y) is normal if and only if for every closed set F and any open set H containing F there exists an open set G such that

$$F \subset G \subset \bar{G} \subset H \qquad \text{(5 marks)}$$