BONDO UNIVERSITY COLLEGE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: SMA *862*
: PARTIAL DIFFERENTIAL EQUATIONS III

Date: …….. December, 2012 ***Time: ……. - ………***

INSTRUCTIONS:

1. This examination paper contains six questions. Answer **any four questions**.
2. Start each question on a fresh page.
3. Indicate question number clearly at the top of each page.

QUESTION ONE (15 marks)

a) Verify that $u = f(x-y) + xg(x-y) + x^2h(x-y)$ is a solution of:

$3u_{xxx} + 3u_{xxy} + 3u_{xyy} + u_{yyy} = 0$ (5 marks)

b) Find the characteristics of the following equation and reduce it to the appropriate standard form and then obtain the general solutions:

$u_{xx} + 4u_{xy} + 4u_{yy} = 0$ (10 marks)

QUESTION TWO (15 marks)

a) Determine for what values of x and y the following equation is elliptic:

$(1+y)u_{xx} + 2(1-x)u_{xy} + (1-y)u_{yy} = u$ (5 marks)

b) Determine the type of the following equation

$u_{xx} + u_{yy} = 0$

And after reducing it to the hyperbolic form, deduce the formula

$$u(x,y) = \frac{1}{2}\phi(x+iy) + \frac{1}{2}\bar{\phi}(x+iy)$$

expressing any harmonic function u as the real part of some analytic function ϕ of complex variable $(x+iy)$ (10 marks)

QUESTION THREE (15 marks)

Derive the D' Alembert's solution of the one - dimensional wave equation

$$u_{tt} = c^2 u_{xx}$$

given the initial conditions

$$\begin{aligned} u(x,0) &= \phi(x) \\ u_t(x,0) &= \psi(x) \end{aligned}$$ (15 marks)

QUESTION FOUR (15 marks)

Solve the two-dimensional wave equation

$$u_{tt} = \lambda^2 (u_{xx} + u_{yy})$$

With the boundary conditions

$$\begin{aligned} u(0,y,t) &= 0, & u(a,y,t) &= 0 \\ u(x,0,t) &= 0, & u(x,b,t) &= 0 \end{aligned}$$

and the initial conditions

$$\begin{aligned} u(x,y,0) &= \phi(x,y) \\ u_t(x,y,0) &= \psi(x,y) \end{aligned}$$ (15 marks)

QUESTION FIVE (15 marks)

Consider the heat conduction in a thin metal bar of length L with insulated sides. Let us suppose that the ends $x=0$ and $x=L$ are held at temperature $u = 0^0 c$ for all time $t>0$. In addition, let us suppose that the temperature distribution at $t=0$ is $u(x,0) = f(x),\ 0 \le x \le L$. Determine the temperature distribution in the bar at some subsequent time $t>0$. (15 marks)

QUESTION SIX (15 marks)

Consider Laplace's equation: $u_{xx} + u_{yy} = 0$, where $u(x, y)$ represents the velocity potential of a fluid particle in a certain domain. Determine $u(x, y)$ inside a unit circle, $x^2 + y^2 < 1$, where its value on the circumference $x^2 + y^2 = 1$, are prescribed. (15 marks)