JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTERS OF SCIENCE IN PURE AND APPLIED MATHEMATICS

COURSE CODE: SMA 839
COURSE TITLE: NUMERICAL ANALYSIS I

DATE: 26/02/2013 **TIME: 2.00-5.00PM**

INSTRUCTIONS:

1. This examination paper contains FIVE questions.
2. Answer **any THREE questions**.

Question1 [20 marks]

Given system of linear equations

$$3x + y + z = 5$$
$$2x + 6y + z = 9$$
$$x + y + 4z = 6$$

(i) Express the system in a matrix form $A\underline{X} = b$ where A is a square matrix and $\underline{X} = [x, y, z]^t$

(ii) Derive the associated Jacobi's and Gauss Seidel's iterative schemes.

(iii) State the conditions under which the Jacobi's iterative scheme will converge to the unique solution of the system .

(iv)Apply eight times Gauss Seidel's iterative scheme on the given system above.

On the same tabulate display the results; $n,\ x_n,\ y_n,\ z_n,\ \delta_n : n = 0, 1, 2, 3, ..., 9$ **where**

$\delta_n = |\underset{\sim}{X}_{n+1} - \underset{\sim}{X}_n|$, $\underset{\sim}{X}_n = [x_n,\ y_n,\ z_n]^t$ with initial vector $\underline{X}_0 = [0, 0, 0]^t$

Comment appropriately on the nature of convergence. [20 marks]

Question2 [20 marks]

Suppose B is an n by n real symmetric matrix of linear operator defined on finite $n-$ dimensional vector space $V^{(n)}$ over field F with n linearly independent eigenvectors, v_i's. Let λ_k be an eigenvalue of B closest to a real number p.

(a) If v_0 is an arbitrary non zero vector in $V^{(n)}$ with a component in the direction of vector v_k,

show that , $\lim_{m\to\infty}\left\{(\lambda_k - p)^m (B - pI)^{-m} v_0\right\} = \alpha_k v_k$. State any assumptions made. [15 marks]

(b) Describe fully the computational procedure for this algorithm; $m = 1, 2, 3,\ 4$.

State for what value of m does the algorithm stop. [5 marks]

Question 3[20 marks]

Given the matrix $A = \begin{bmatrix} 10 & 7 & 4 & 1 \\ 7 & 11 & 1 & 2 \\ 4 & 1 & 5 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

(a)Determine and describe briefly the procedure for power method. [6 marks]

(b)Apply six times the power method, to approximate the dominant eigenvalue λ_1, of A.

Obtain $\underline{v}_1$, the corresponding eigenvector. Tabulate the results; $n,\ \alpha_n,\ \underline{x}_n,\ \hat{\lambda}_n\ ,\ \lambda_1 : n = 1, 2, 3, ..., 6$

Take the initial vector, $\underline{v}_0 = [1, 1, 1, 1]^t$ [14marks]

Question 4[20 marks]

Consider the system of nonlinear equations

$$f(x,y) = x^2 + y^2 - 1 = 0$$

$$g(x,y) = 2y^2 - 2x^2 - 1 = 0$$

(a) Derive the improved Newton's iterative scheme

$$x_{n+1} = x_n - \frac{g(x_n, y_n) + f(x_n, y_n)}{4x_n}$$

$$y_{n+1} = y_n + \frac{f(x_{n+1}, y_n) - g(x_{n+1}, y_n)}{-4y_n}$$, for the system. [5marks]

(b) Apply six times the improved Newton's iterative scheme to obtain the approximate solution of the system. On the same table display the results; $n, x_n, y_n, f(x_n, y_n), g(x_n, y_n)$.

Take the initial root as $(x_0, y_0) = (1,\ 3.5)$ [15 marks]

Question.5 **[20 marks]**

(a) Determine the Padé rational approximation to $f(x) = e^{-x}$ of degree 5 with n=3,and m=2 of the form $r_{3,2}(x)$.

On the same table, display the values of : x , e^{-x} , $r_{3,2}(x)$, $|e^{-x} - r_{3,2}(x)|$: $x =, 0.2, 0.4, 0.6, 0.8, 1.0$

Comment appropriately on the nature of the accuracy of the rational function $r_{3,2}(x)$.[10marks]

(b) Using the Newton's Divided Difference Table below , construct an interpolating polynomial $p_4(x)$ t**o approximate** $f(1.3)$. **If** $f(x) = \ln x$ **,compute the relative error.** [10 marks]

Newton's Divided Difference Table

x_i	$f[\ ]$	$f[\ ,\ ]$	$f[\ ,\ ,\ ]$	$f[\ ,\ ,\ ,\ ]$	$f[\ ,\ ,\ ,\ ]$
1	0.00000				
		.81094			
1.5	.40547		−.25912		
		.61660		0.09416	
1.75	.55962		−.16496		−.05980
		.53412		.08818	
2	.69315		−.20023		
		.66427			
1.1	0.095531				