JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES
UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)
4TH YEAR 2ND SEMESTER 2019/2020 ACADEMIC YEAR
MAIN REGULAR

COURSE CODE: SPH 403

COURSE TITLE: QUANTUM MECHANICS II

EXAM VENUE: **STREAM: EDUCATION**

DATE: **EXAM SESSION:**

TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Attempt question ***1(Compulsory)*** and any other ***TWO*** questions in **section B.**

SECTION A

QUESTION 1 (30 MARKS)

a)

i. State any **TWO** postulates of Quantum mechanics **(2 marks)**

ii. Show that if two wave functions ψ_1 and ψ_2 satisfy the time-dependent Schroedinger equation, then their superposition $\alpha\psi_1 + \beta\psi_2$ in which α and β are constants is also a wave function satisfying the same Schroedinger equation. **(3 marks)**

b) Write down the TWO main mathematical properties of the quantum wave function that formed the basis for Dirac's formulation of Quantum mechanics. **(2 marks)**

c) Distinguish between spin-orbit coupling and Schroedinger picture. **(2 marks)**

d) Determine the THREE components of orbital angular momentum in cartesian coordinate system. **(3 marks)**

e) Calculate the commutation relation $\left[\hat{L}_x, \hat{L}_y^2\right]$ **(4 marks)**

f) Derive the energy spectrum of a quantized linear harmonic oscillator in a number state $|n\rangle$ given the Hamiltonian $\hat{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$ where the symbols have their usual meaning **(4 marks)**

g) Distinguish between time-independent perturbation theory and time-dependent perturbation theory. **(2 marks)**

h) State the selection rules for allowed transitions in hydrogen atom. **(2 marks)**

i) Write down the Classical and corresponding Quantum mechanical form of the Hamiltonian of a two particle system **(2 marks)**

j) List the state vectors $|u\rangle;|d\rangle$ and use them to determine the spin state transition operators of a two-state system, $\hat{S}_+$; S_- **(4 marks)**

SECTION B

Attempt any TWO questions in this section.

QUESTION 2 (20 MARKS)

a) Determine the expression for the ladder operator $\hat{L}_+$ of the angular momentum given

$\hat{L}_x = i\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cos\phi\cot\theta\frac{\partial}{\partial\phi}\right)$ and $\hat{L}_y = i\hbar\left(-\cos\phi\frac{\partial}{\partial\theta} + \sin\phi\cot\theta\frac{\partial}{\partial\phi}\right)$ **(5 marks)**

b) The number state vector $|n\rangle$ of a quantized harmonic oscillator satisfies the state transition algebraic relations $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$; $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$.

(i) Identify the operators $\hat{a}$ and $\hat{a}^\dagger$ **(2 marks)**

(ii) By evaluating $[\hat{a}, a^\dagger]|n\rangle$, show that $[\hat{a}, \hat{a}^\dagger] = 1$ **(5 marks)**

(iii) Determine the uncertainty in the measurement of the quadrature component $\hat{x}_1$ in the number state $|n\rangle$, using the definition $\hat{a} = \hat{x}_1 + i\hat{x}_2$; $\hat{a}^\dagger = \hat{x}_1 - i\hat{x}_2$ **(8 marks)**

QUESTION 3 (20 MARKS)

a) Explain the Heisenberg picture of Quantum mechanics, hence derive the Heisenberg's equation of motion. **(8 marks)**

b) Show that the spin-up and spin-down state vectors of an electron satisfy the orthonormality relations $\langle u|u\rangle = \langle d|d\rangle = 1$; $\langle u|d\rangle = \langle d|u\rangle = 0$ **(3 marks)**

c) Angular momentum operators can be defined by the following matrices.

$$\hat{L}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};\ \hat{L}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix};\ \hat{L}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

. Determine the commutation brackets $[\hat{L}_x, \hat{L}_y]$, $[\hat{L}_y, \hat{L}_z]$ and $[\hat{L}_z, \hat{L}_x]$ in terms of $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$

(9 marks)

QUESTION 4 (20 MARKS)

a) Using $u(r) = rR(r)$, the radial equation for a one-electron atom is obtained in the form $\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2\mu r^2} - \frac{Ze^2}{4\pi\varepsilon_0 r}\right)u(r) = Eu(r)$ where the symbols have their usual meanings.

(i) By completing the square of the effective potential, determine the

quantized orbit energy in the form $E_{l+1} = \frac{-\mu Z^2 e^4}{2(4\pi\varepsilon_0)^2\hbar^2(l+1)^2}$. **(5 marks)**

(ii) Show that the radial equation can be factorized in the form

$\left(\frac{d}{dr} + K_{l+1}(r)\right)\left(-\frac{d}{dr} + K_{l+1}(r)\right)u(r) = \frac{2\mu}{\hbar^2}(E - E_{l+1})u(r)$ where the parameter $K_{l+1}(r)$

must be defined in the derivation. **(5 marks)**

b) A particle moves in the one-dimensional potential defined by $V(x) = \begin{cases} V_0 \cos\left(\frac{\pi x}{2a}\right) & |x| \le a \\ \infty & |x| > a \end{cases}$

By treating the potential as a perturbation, obtain the first order energy correction, given that the unperturbed eigen function is

$$u_n = \begin{cases} \frac{1}{\sqrt{a}}\cos\left(\frac{n\pi x}{2a}\right) & n = odd \\ \frac{1}{\sqrt{a}}\sin\left(\frac{n\pi x}{2a}\right) & n = even \end{cases}.$$

(10 marks)

QUESTION 5 (20 MARKS)

a) A two-level system described by the wave function $\psi(t) = c_a(t)\psi_a e^{-iE_a\frac{t}{\hbar}} + c_b(t)\psi_b e^{-iE_b\frac{t}{\hbar}}$ experiences a time-dependent perturbation. Suppose the system is in state ψ_a intially, derive the expressions for the first order approximations of probability amplitudes, $c_a^1(t)$ and $c_b^1(t)$. **(12 marks)**

b) If the perturbation in 5 (a) above is of the form $H_{ab}^I = V_{ab}\cos\omega t$, show that the transition probability is given by $P_{a\to b} = \frac{|V_{ab}|^2}{\hbar^2}\frac{\sin^2(\omega_0 - \omega)\frac{t}{2}}{(\omega_0 - \omega)^2}$ **(8 marks)**