JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESIT 2020/2021 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAS 103

COURSE TITLE: INTRODUCTION TO PROBABILITY THEORY

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) In a club there are 20 males and 15 females. Four tickets will be distributed among 4 members to enjoy drama. What is the probability that the tickets will be given to
   i. 4 females
   ii. Males and females equally
   iii. At least a male (9marks)

b) The discrete random variable X has the probability function

$$P(X = x) = \frac{x}{15}; x = 1,2,3,4,5$$

Find the distribution function of X (6 marks)

c) Two unbiased dice are thrown. Let X be the random variable representing the sum of the upper faces of the dice. Find the expected value of the sum of the upper faces (7 marks)

d) From past experience, it is known that 3 out of 5 war planes return safely after attacking the opponent's territory. One day six such war planes left for war. Find the probability that out of the six planes;
   i. 3 returned safely
   ii. At best two returned safely
   iii. At least 3 returned safely (8marks)

QUESTION TWO (20 MARKS)

a) The average printing mistakes per page is 1. Find the probability that in a randomly selected page;
   i. There are 5 mistakes
   ii. There are at best two mistakes
   iii. There are at least 3 mistakes
   iv. Suppose 5 pages are selected, what is the probability that there will be more than two mistakes? (10marks)

b) The marks of 500 candidates in an examination are normally distributed with mean of 45 marks and a standard deviation of 20 marks.
   i. Given that the pass mark is 41, estimate the number of candidates who passed the examination.
   ii. If 5% of the candidates obtain a distinction by scoring x marks or more, estimate the value of x
   iii. Estimate the number of candidates who scored more than 48 but less than 60 marks (10 marks)

QUESTION THREE (20 MARKS)

a) The random variable X takes integer values only and has p.m.f

$$P(X = x) = \begin{cases} Kx & , x = 1,2,3,4,5 \\ K(10 - x), & x = 6,7,8,9 \end{cases}$$

Find

i. K

ii. $Var\ (X)$

iii. $E(2X - 3)$

iv. $Var\ (2X - 3)$ (10 marks)

b) The continuous random variable X has p.d.f $f(x)$ where

$$f(x) = \begin{cases} c(x+2)^2\ , & -2 \le x < 0 \\ 4c\ , & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

Find

i. the value of c

ii. $P(-1 \le X \le 1)$

iii. $P(X > 1)$ (10 marks)

QUESTION FOUR (20 MARKS)

a) For the events C and D, $P(C) = 0.55$, $P(CUD) = 0.9$, $P(C \cap D) = 0.25$. Find,

i. $P(D)$

ii. $P(D^C \cap C)$

iii. $P(C^C \cap D)$

iv. $P(D^C \cap C^C)$

v. $P(D/C)$ (10marks)

b) In a class of 80 students 50 know English, 55 know French and 46 know German language. 37 students know English and French, 28 students know French and German, 25 know English and German. 12 students know all the three languages. Find the probability that :

i. A student knows only English

ii. A student knows only one language

iii. A student knows German given he knows French (10marks)

QUESTION FIVE (20 MARKS)

a) The percentage of people exposed to bacteria who become ill is 20%. Assume that the people are independent. If 1000 people are exposed to bacteria, approximate;

i. The probability that more than 225 become ill

ii. The probability that between 175 and 230 become ill

iii. The value such that the probability that the number of people that become ill exceeds this value is 0.01 (10 marks)

b) Suppose X is a binomial random variable with $n = 200$, $p = 0.3$

i. Approximate the probability that $x > 52$ using a standard normal random variable.

ii. Approximate $P(48 < X < 70)$ using a standard normal random variable.

iii. Find the probability that a variable X is either less than 50 or greater than 65. (10marks)