JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

YEAR ONE SEMESTER ONE EXAMINATION (Special Resit) 2020

SMA 101: ANALYTIC GEOMETRY

INSTRUCTION: Answer Question ONE and ANY other TWO questions.

QUESTION ONE (COMPULSORY)

a) Define the Conic sections below
   i)Parabola
   ii) Ellipse (2 marks)

b) Determine the the distance between two parallell lines $y = 4x + 10$ and the line passing through the points (0,0) and (-1,-4) (4 marks)

c) A line L_1 has an equation $y = -2x + 6$. Calculate the acute angle between L_1 and L_2whose equation is $3y + 2y + 6 = 0$ (4 marks)

d) Calculate the area of a circle which passes through (4, 7), (-13, 0) and (11,0). (Give your area in terms of π) (7 marks)

e) Convert the following polar coordinates in to Cartesian coordinates
   (i)(-4, 200^0) (ii) $(6, -\frac{\pi}{4})$ (4 marks)

f) Use the third order matrix determinant to determine the equation of a line passing through the points (-5, -2) and (15, 3) giving your answer in double intercept form, hence declaring the intercepts. (4 marks)

g) Determine the centre and area of an Ellipse bounded by the lines lines $y = 8$, $y = -2$ $x = 3$ and $x = -3$. (5 marks)

QUESTION 2(20 MARKS)

a) The equation of an ellipse is given by $72x^2 + 50y^2 - 432x + 400y - 352 = 0$

Find on the *xy* plane

(i)The centre of the ellipse (4 marks)

(ii)The coordinates of the vertices (2 marks)

(iii) The foci (2 marks)

(iv) The eccentricity (1 mark)

(vi) The directrices (2 marks)

(vii) The area of the ellipse (3 marks)

b) A second degree curve is represented by the equation $x^2 - 2xy + y^2 - 16x - 48y = 0$. By eliminating the cross product term identify the conic section hence give its equation on the $x'y'$ plane and state the equation of the axis. (6 marks)

QUESTION 3(20 MARKS)

a) The equation of a hyperbola is given as $3x^2 - 12x - \frac{4}{3}y^2 - 8y - 12 = 0$. Find

(i) The coordinate of the centre. (4 marks)

(ii) The foci of the hyperbola on the *xy* plane. (2 marks)

(iii) The vertices on the *xy* plane. (2 marks)

(iv) The asymptotes on the $x'y'$ plane and on the *xy* plane. (4 marks)

(vi) The eccentricity (1 mark)

(vii) The direcrices on the $x'y'$ plane and on the *xy* plane. (3 marks)

b) Give the cartesian equation of the following pairs of parametric equations

(i) $x = t - t^2$, $y = t^2 + t^3$ (2 marks)

(ii) $x = \dfrac{2t}{1+t^3}$, $y = \dfrac{2t^2}{1+t^3}$ (2 marks)

QUESTION 4(20 MARKS)

a)Find the parametric equation of the following Cartesian equations

(i) $x^3 + y^3 = 3xy$ (2 marks)

(ii) $xy = x - y$ (2 marks)

(iii) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ (2 marks)

b) (i)A conic section has the equation $3x^2 + 4\sqrt{3}xy - y^2 = 7$. Rotate the axes of the conic section by eliminating the cross product term. (8 marks)

(ii)Identify the conic section on the new $x'y'$ plane (2 marks)

(iii) Give the coordinates of the focus/foci of the conic on the $x'y'$ plane. (2 marks)

(iv) Find the eccentricity of the conic section (2 marks)

QUESTION 5(20 MARKS)

a) Sketch and give the name of the polar curves $r = 1 + 4\cos\theta$ (6 marks)

b) Identify the conic sections given below (6 marks)

$i) 4x^2 - 4xy + y^2 - 5\sqrt{5}x + 5 = 0$

$ii) 3x^2 - 4\sqrt{3}xy - y^2 = 24$

$iii) r = \dfrac{4}{2 - 2\cos\theta}$

$iv) r(8 + 6\sin\theta) = 0$

c)A parabola has the *y*- intercepts -1 and 2 while the *x*-intercept is 4 (8 marks)

Find

(i) The equation of the parabola

(ii) The equation of the axis of the parabola

(iii) The vertex and focus of the parabola

(iv) The equation of the directrix of the parabola