JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESITS EXAMINATIONS, NOVEMBER 2020

COURSE CODE: SMA 102

COURSE TITLE: CALCULUS I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Explain why the limit below do not exist:

$\lim_{x\to 0}\frac{x}{|x|}$ (3 marks)

b) Find $\lim_{x\to 4}\frac{x^2-x-12}{x-4}$. (4marks)

c) Determine the point of discontinuity (if any) of the function $f(x)$

$f(x)=\frac{x^2-4}{x+2}$.

If the discontinuity is removable, define the function to make it continuous. (4 marks)

d) Using the definition, find the derivative $f'(x)$ of the function $f(x)=2x^2-3x+5$ (4 marks)

e) Find the critical numbers of $f(x)=x^3-5x^2-8x+3$ and determine whether they yield relative maximum, relative minimum or inflection points (5 marks)

f) Find the derivative of the function

$y=\frac{x\ln x}{1+\ln x}$ (5 marks)

g) Given that $f(x)=\cos x$, prove that $f'(x)=-\sin x$ (5 marks)

QUESTION TWO (20 marks)

a) Prove that

$\frac{d}{dx}(uv)=u\frac{dv}{dx}+\frac{du}{dx}v$ (6 marks)

b) Given that $f(x)=\frac{2x+1}{x^2-1}$, find $f'(x)$ (4 marks)

c) Find the derivative of $y=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$ (6 marks)

d) Find the derivative of the function

$f(x)=\left(\frac{1+\sin x}{\cos x}\right)^{-1}$ (4 marks)

QUESTION THREE (20 marks)

a) If $x^3+y^3=16$, find $\left.\frac{d^2y}{dx^2}\right|_{(3,3)}$ (6 marks)

b) Find the first and second order derivatives of:

$s=\frac{12}{t}-\frac{4}{t^3}+\frac{1}{t^4}$

with respect to t. (5 marks)

c) If $x=a\cos\theta$, $y=b\sin\theta$, show that

$$\frac{d^2y}{dx^2}=-\frac{b}{a^2}\cos ec^3\theta \quad \text{(9 marks)}$$

QUESTION FOUR (20 marks)

i. The position of a particle which moves along a straight line is defined by the

relation $x = t^3 - 4.5t^2 - 12t + 36$, where x is expressed in feet and t in seconds.
Determine:

a. the time at which the velocity will be zero, (3 marks)
b. the position and distance traveled by the particle at that time, (4 marks)
c. the acceleration of the particle at that time, (3 marks)
d. the distance traveled by the particle from $t = 3s$ to $t = 5s$. (4 marks)

ii. For several weeks, highway department has been recording the speed of freeway traffic flowing past a certain downtown exit. The data suggest that between 1.00 and 6.00 P.M. on a normal weekday, the speed of the traffic at the exit is approximately $S(t) = t^3 - 10.5t^2 + 30t + 20$ miles per hour, where t is the number of hours past noon. At what time between 1.00 and 6.00 P.M. is the traffic moving the fastest, and at what time is it moving the slowest? (6 marks)

QUESTION FIVE (20 marks)

i. Use Logarithmic differentiation to find the derivative of y with respect to x

$$y = \frac{\sin^2 x \tan^4 x}{\left(x^2 + 1\right)^2}$$ (7 marks)

ii. Find the derivative of y with respect to x, given that

$\ln xy = e^{x+y}$ (6 marks)

iii. Find the derivative of y with respect to x, where

$y = L\text{og}_2\left(e^{-x} \cos \pi x\right)$ (7 marks)