JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITYDRAFT EXAMINATION FOR BSc/BEd IN MATHEMATICS

SUPPLEMENTARY/SPECIAL

2nd YEAR 1st SEMESTER 2019/2020ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA201
COURSE TITLE: LINEAR ALGEBRA II

EXAM VENUE: AUDITORIUM **STREAM: BSc Y2S1**

TIME: 2 HOURS **EXAM SESSION:**

Instructions:

Answer question1 and any other two questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

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QUESTION ONE [30 MARKS] COMPULSORY

a) Define the following terms in relation to linear spaces

i) Span (2mk)

ii) Dimension (2mk)

b) Show that the eigenvalues of the matrix $A = \begin{bmatrix} 51 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & -31 \end{bmatrix}$ are ;51, 11, -31 (10mks)

c) Let $T: \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^3$ be a linear operator defined by

$T(x_1, x_2, x_3) = (2x_1 - x_2 - x_3,\ x_1 - x_3,\ -x_1 + x_2 + 2x_3)$. Find the matrix B associated with T with respect to the standard ordered basis (8mks)

d) Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

Find all the eigenvalues of A and the corresponding eigenvectors (8mks)

QUESTION TWO (20 MARKS)

a) Let $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 6 \\ -2 & 14 \end{bmatrix}$. Show that $(AB)^{-1} = B^{-1}A^{-1}$ (10mks)

b) Consider the following bases $B = \{(1,0), (0,1)\}$ and $B' = \{(1,2), (2,3)\}$ for $\boldsymbol{R}^2$. If $T : \boldsymbol{R}^2 \rightarrow R^2$ is a linear transposition defined by $T(x_1, x_2) = (x_1 + 7x_2,\ 3x_1 - 4x_2)$.

i) Find A, the matrix of representation of T with respect to B

ii) Find M, the matrix of representation of T with respect to B' (10mks)

QUESTION THREE (20 MARKS)

Let $T : \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^3$ be a linear operator from a vector space $\boldsymbol{R}^3$ to itself defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 - 4x_3 \\ x_1 - x_3 \\ -x_1 + x_2 + 2x_3 \end{bmatrix}$

i) Obtain M the matrix of the linear operator T (8mks)

ii) Find the characteristic polynomial of the operator T (6mks)

iii) Find the eigenvalues of T and their corresponding eigenvectors (6mks)

QUESTION FOUR (20 MARKS)

a) Verify that the set $S = \{(x,\ 3x):\ x \epsilon \boldsymbol{R}\ \}$ is a subspace of $\boldsymbol{R}^2$ (4mks)

b) i) State Cayley Hamilton theorem (3mks)

ii) Give matrix $A = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix}$,show that both A , $\boldsymbol{A}^{-1}$ satisfy Cayley Hamilton theorem .

(8mks)

c) i) Define the term kernel (2mk)

ii) If $T: U \to V$ is a linear mapping, show that the kernel of T is a subspace of U (3mks)

d) Find the matrix of linear mapping $T\colon\ P_3\ \to\ P_1$ given by $T(f) =\ f'' +\ f'''$ (4mks)

QUESTION FIVE (20 MARKS)

a) Define the term orthogonality of vectors in a vector space W (4mks)

b) Let $P = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{pmatrix}$ be a real square matrix.

Prove that P is orthogonal hence and find $\hat{P}$ the orthonormalized form of P and $\hat{P}^{-1}$.**[10marks]**

c) If V is a linear space of all functions of the form $f(t) =\ c_1 \cos t +\ c_2 \sin t$, where c_1 and c_2 are arbitrary constants,

Find the matrix of linear transformation $T(f) =\ f''' + af'' + bf'$ with respect to the basis $\cos t$, $sin\ t$ where a and b are arbitrary constants (6mks)