JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE/ ARTS

SPECIAL RESIT 2020/2021 ACADEMIC YEAR

REGULAR (MAIN) SPECIAL RESIT

COURSE CODE: SMA 402

COURSE TITLE: MEASURE THEORY

EXAM VENUE: **STREAM: BED SCI/ARTS**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

<u>Instructions:</u>

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

a) Define upper and lower Lebesgue sums (4 marks)
b) Show that the measure of a singleton set is zero. (4 marks)
c) Give two examples $\sigma - Algebra.$ (2 marks)
d) State and prove Fatou's Lemma (10 marks)
e) Show that $A \cup B \in \mathcal{M}$ and $A \cap B \in \mathcal{M}$, where $\mathcal{M}$ is Lebesgue Measurable Set. (10 marks)

QUESTION TWO

a) Describe the following terms: (4 marks)
i.) Almost everywhere concept
ii.) Complete measure space
b) Let $g: M \to \mathbb{R}$ be $\mathfrak{x}$- measurable. Then show that g^2 is $\mathfrak{x}$- measurable. (5 marks)
c) State and prove Monotone Convergence Theorem. (11 marks)

QUESTION THREE

a) Show that the length the Outer Measure of interval is equal to the length of the interval, $I \in [a, b]$. (10 marks)
b) Outline the difference between Lebesgue Integral and Riemann integral. (4 marks)
c) Describe the following: (6 marks)
i) Lebesgue Dominated Convergence Theorem
ii) Fatou's Lemma

QUESTION FOUR

a) Describe a Measurable Space. (3 marks)
b) Show that, if $S, T \subseteq \mathbb{R}$ and $S \subseteq T$ then $\mu^*(S) \leq \mu^*(T)$ $i.e$ μ^* is monotone. (3marks)
c) Prove that any non-degenerate interval of $\mathbb{R}$ is uncountable. (10marks)
d) Define Borel Measurable Subsets of $\mathbb{R}$ and give two examples. (4 marks)

QUESTION FIVE

a) Show that every bounded Riemann integral functions over $[a, b]$ is Lebesque integrable and the two integrals are the same. (10 marks)
b) Let $(X, \mathfrak{x}, \mu)$ be a Measure Space and $f, g \in M^+(X, \mathfrak{x})$ and k a non-negative real constant. Prove that
$\int (f + g) du = \int f\, du + \int g du$ and $\int kf du = k \int f du.$ (10 marks)