Int. J. Math. And Appl., 9(4)(2021), 25–34
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Jump Diffusion Logistic Brownian Motion With Dividend Yielding Asset

M. O. Opondo[1,*], D. B. Oduor[1] and F. Odundo[1]

1 School of Mathematics and Actuarial Science, Jaramogi Oginga Odinga University of Science and Technology, Bondo, Kenya.

Abstract: Jump diffusion processes have been used in modern finance to capture discontinuous behavior in asset pricing. Logistic Brownian motion for asset security prices shows that naturally asset security prices would not usually shoot indefinitely due to the regulating factor that may limit the asset prices. Geometric Brownian motion can not accurately reflect all behaviors of stock quotation therefore, Merton who was involved in the process of developing the Black-Scholes model came up with Merton jump model superimposed on Geometric Brownian motion without considering the dividend yielding rate of the asset. Therefore in this paper, we have derived the price of dividend yielding asset that follows logistic Brownian motion with jump diffusion process. This study uses the knowledge of Geometric Brownian Motion and logistic Brownian motion with Heaviside's Cover-up Method to develop the price of dividend yielding asset that follows logistic Brownian motion with jump-diffusion process.

MSC: Primary 91GXX, 91G50, 62 P05, 97M30.

Keywords: Geometric Brownian motion, Logistic Brownian motion, Jump diffusion, dividend yielding asset, Stochastic Process, Wiener process, Ito's Process.

1. Introduction

In Financial Mathematics, pricing of assets has been greatly applied where options are valued in terms of derivatives and securities. Option Pricing Models are mathematical models that use certain variables such as Underlying stock or index price, Exercise price of the option, Expiry date of the option, Expected dividends (in cents for a stock, or as a yield for an index), Expected risk free interest rate, Expected volatility of the underlying stock or index over the life of the option to calculate the theoretical value of an option. The theoretical value of an option is an estimate of what an option should worth using all known inputs. In other words, option pricing models provide us with a fair value of an option. Knowing the estimate of the fair value of an option, financial professionals could adjust their trading strategies and portfolios.

Therefore, option pricing models are powerful tools for finance professionals involved in options trading. Pricing of options and assets is partly derived from the interplay of demand and supply and partly from theoretical models. Standard Black-Scholes equation is derived under some strict assumptions such as Constant volatility, Efficient markets (underlying asset follows a geometric Brownian motion), the underlying stock does not pay dividends during the option's life, Interest rates constant and known, Lognormally distributed returns, European-style options, No commissions and transaction costs and markets are perfectly liquid. These assumptions are sometimes not applicable in real market world. Option pricing depends on the hypothesis that the dynamics of the underlying asset is sold if its price decreases and bought if its price increases

* *E-mail: markopondo1994@gmail.com*

in a perfectly liquid market. A standard model for price of stock as a function of time S(t) evolves according to geometric Brownian motion.

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t), \tag{1}$$

where μ is the rate of growth of the asset(percentage drift), σ is the volatility (percentage volatility) and $dW(t)$ is the wiener process(Brownian motion). This model is based on the idea that prices appear to be the previous price plus some random change and that these price changes are independent, prices are taken to follow some random walk-type behavior. This is the basis for including the stochastic function which is the Wiener process $dW(t)$. The demand and supply curves are also used to determine the quantity and price at which assets are bought and sold. The supply curve shows what the quantities the sellers are willing and able to sell at various prices whereas demand curve shows the quantities the consumers are willing and able to buy at different prices. The interplay between supply and demand brings in what is called market equilibrium. This is the situation where there is no tendency for change in asset price and quantity. In stock markets the price of an asset is assumed to respond to excess demand and is expressed as $ED(S(t)) = QD(S(t)) - QS(S(t)$, where ED(S(t) is excess demand, $QD(S(t))$ are quantities demanded and $QS(S(t))$ are quantities supplied at a given time t and price S(t). The market structure with forces of demand and supply experience upward and downward shifts until a state of market equilibrium is achieved.

A lot of literature dealing with pricing and hedging of contingent claims are based on a basic assumption that the asset's price follows a geometric Brownian motion. Emperical studies have shown that the models based on Geometric Brownian motion may be limited in describing stock's price evolution hence inducing mispricing through overestimation or underestimation. To be able to produce more accurate option pricing, the jump diffusion model was introduced by Merton [19]. The jump diffusion model unlike the famous Black- Scholes models [3] do not make the same assumptions of normally distributed logarithmic returns. Stock prices may change due to the general economic factors such as demand and supply, changes in economic outlook and capitalization rates. These brings about small or marginal movements in stock's price hence modeled by a Geometric Brownian motion. On the other hand, the stock's price may fluctuate due to announcement of some important information causing over-reaction or under-reaction of the asset prices due to good and/or bad news. This information may emanate from the firm or industry. Such information that arrives at discrete points in time can only be modeled by a jump process.

Volatility is a measure of how uncertain we are about the future of stock price, hence the estimation of volatility is crucial for implementation and valuation of asset and derivative pricing. A dividend is a payment made by a corporation to its shareholders, usually as a distribution of profits. When a corporation earns a profit or surplus, the corporation is able to re-invest the profit in the business (called retained earnings) and pay a proportion of the profit as a dividend to shareholders. A logistic equation that is used in natural science was used to develop a logistic geometric Brownian motion with a price of dividend yielding assets. In this paper, we have focused on the issues related to jump diffusion model for the price of dividend yielding assets that follows logistic Brownian motion in financial market.

2. Preliminaries

In this section, we recall some key basic concepts, methodologies and the result that are fundamental in the study:

2.1. Stochastic Process

Any variable whose value changes over time in uncertain way and where we only know the distribution of the possible values of the process at any point of time is said to follow a stochastic process, Hence it obeys laws of probability. Mathematically, a stochastic process $X = [X(t); t \in (0; \alpha)]$ is a collection of random variables such that for each t in the index set $(0; \alpha)$,X(t) is a random variable where X(t) is the state of the process at time t. A discrete time stochastic is the one where the value of the variable can only change at a certain fixed points in time. On the other hand continuous time stochastic, change can take any value within a certain range.

2.1.1. Wiener Process

The Wiener process W(t), also called Brownian motion, is a kind of Markov stochastic process which is in essence a series of normally distributed random variables,and for later time points, the variances of these normally distributed random variables increase to reflect that it is more uncertain (thus more difficult) to predict the value of the process after a longer period of time. Instead of assuming $W(t) \sim N(0; t)$, which cannot support algebraic calculations, the Wiener process W(t) is introduced. $\Delta W \equiv \epsilon\sqrt{\Delta t}$ (change in a time interval Δt) where $\epsilon \backsim N(0, 1)$ and ΔW follows a normal distribution. i.e $E[\Delta W] = 0$ and $Var[\Delta W] = \Delta t$ which gives a standard deviation given as $std[\Delta W] = \sqrt{\Delta t}$ and

$$W(T) - W(0) = \sum_{i=1}^{n} \epsilon_i \sqrt{\Delta t} = \sum_{i=1}^{n} \Delta W_i \quad where \quad n = \frac{T}{\Delta t}$$

where $W(T)-W(0)$ also follows a normal distribution where $E[W(T)-W(0) = 0$ and $Var[W(T)-W(0)] = n.\Delta t = T$ which gives a standard deviation given by $std[W(T) - W(0)] = std[W(T) = \sqrt{T}$ As $n \longrightarrow \infty$, Δt converges to 0 and is denoted as dt,which means an infinitesimal time interval. Correspondingly, $\Delta W$is denoted as dW. In conclusion, dW is invented to simplify the representation of a series of normal distribution which is a Wiener Process. The properties of Wiener process $W(t)$ for $t \geq 0$ are; has a normal increment i.e $W(t) - W(s) \backsim N(0, 1)$, The increments are independent i.e W(t) - W(s) and $W(\mu)$ are independent , for $\mu \leq s < t$ and There exists a continuity of the path i.e W(t) is a continous function of t.

2.1.2. Generalised Wiener Process

The basic Wiener process, dW that has been developed so far has a drift rate of zero and a variance rate of 1.0. the drift rate of zero means that the expected value of Z at any future time is equal to its current value. The variance rate of 1.0 means that the variance of the change in Z in time interval of length T equals T. A generalised Wiener process for a variable X can be defined in terms of dW as $dX = adt+bdW$ Where mean rate a and variance rate b are constants, adt is the expectation of dX and bdZ is the addition of noise or variability to the path followed by X, while b is the diffusivity. In a small interval Δt, the change in the value of $X; \Delta X$ is of the form $\Delta X = a\Delta t + b\varepsilon\Delta t$ where ε is a random variable drawing from standardised normal distribution thus the distribution of ΔX is thus, Standard deviation of $\Delta X = b\sqrt{\Delta t}$ Hence $\Delta X \backsim N(a\Delta t; b\sqrt{\Delta t})$ Similar argument to those given for a Wiener process show that the change in the value of X in any time interval T is normally distributed with mean of change in $X = aT$, Variance of change in $X = b^2T$ and Standard deviation of change in $X = b\sqrt{T}$ Hence $\Delta X \backsim N(aT; b\sqrt{T})$.

2.1.3. Ito's Process

This is the generalised Wiener process in which the parameters a and b are functions of the value of the underlying variable X and time t [20]. An Ito's process can be written algebraically as; $dX = a(X; t)dt + b(X; t)dW$. Both the expected drift rate and variance rate of an Ito's process are liable to change over time. In a small time interval between t and $t + \Delta t$, the changes from X to $X + \Delta X$, is expressed as

$$\Delta X = a(X; t)\Delta t + b(X; t)\epsilon\sqrt{\Delta t}$$

This relationship involves a small approximation. It assumes that the drift(mean) rate and variance rate of X remain constant, equal to $a(X; t)\Delta t$ and $b^2(X; t)\Delta t$ respectively during the interval between t and $t+\Delta t$ hence $\Delta X \backsim N(a(X; t)\Delta t+ b(X; t)\sqrt{\Delta t})$.

2.1.4. Geometric Brownian motion

A specific type of Ito's process is the geometric brownian motion of the form; $dX = aXdt + bXdW$ where $a(X,t) = aX$ and $b(X,t) = bX$ (Black Scholes 1973), in the equation above, the geometric Brownian motion has been applied in stock pricing and is given as $dS = \mu Sdt + \sigma SdW$, where S is the stock price, μ is the expected rate of return per unit time and σ is the volatility of stock price [20]. The volatility equation is expressed as; $\frac{dS}{S} = \mu dt + \sigma dW$ This model is the most widely used model of stock price behavior. A review of this model gives a discrete time model, $\frac{\Delta S}{S} = \mu\Delta t + \sigma\epsilon\sqrt{\Delta t}$, where ΔS is the change in stock price S within a small interval of time Δt and ϵ is a random variable drawn from standardized normal distribution with mean zero and standard deviation 1. Hence in a short time Δt, the expected value of return is $\mu\Delta t$ and the stochastic component of the return is $\sigma\epsilon\sqrt{\Delta t}$. The variance of the fractional rate of return is $\sigma^2\Delta t$ and $\sigma\sqrt{\Delta t}$ is the standard deviation. Therefore $\frac{\Delta S}{S}$ is normally distributed with mean $\mu\Delta t$ and standard deviation $\sigma\sqrt{\Delta t}$ or $\frac{\Delta S}{S} \sim N(\mu\Delta t, \sigma\sqrt{\Delta t})$

2.2. Jump Diffusion Process

Jump diffusion processes have been used in modern finance to capture discontinuous behavior in asset pricing [25]. Merton [16] who was also involved in the process of developing the Black-Scholes model [3] came up with Merton jump model as a better estimation of option prices in a precise way. The Merton model has the same assumptions as those of Black-Scholes except that for how the asset price is modeled. This model where the asset price has jumps superimposed upon a geometric Brownian motion is given by

$$dS(t) = S(t)(\mu - \lambda k)dt + S(t)\sigma dW + S(t)(q-1)dq(t), \tag{2}$$

where μ is expected return from the asset, λ is the rate at which jumps happen and $k = \epsilon(q-1)$ is the average jump size measured as a proportional increase in the asset price. $(q-1)$ is the random variable percentage change in the asset price if the poisson event occurs, ϵ is the expectation operator over the random variable q, dW(t) is the standard Wiener process and $dq(t)$ is the independent Poisson process generating the jumps. μdt is adjusted by λkdt in the drift term to make the jump part unpredictable innovation Merton [16].

2.3. Geometric Brownian Motion with Jump Diffusion process

Black Scholes [3] approach to option price estimations and option trading brought about a great breakthrough in financial mathematics . In their assumptions, the price of an option follows a geometric Brownian motion. From empirical study, geometrical Brownian motion cannot accurately reflect all behaviors of the stock quotation. Merton who was also involved in the process of developing the Black-Scholes model came up with Merton jump model (1976) as a better estimation of option prices in a precise way. The model where the asset price has jumps superimposed upon a geometric brownian motion is given by;

$$dS_t = S_t(\mu - \lambda k)dt + S_t\sigma dW_t + S_t dq_t \tag{3}$$

where; μ is the expected return from the asset(asset price growth rate); λ is the rate at which jumps happen; $k = \epsilon(q-1)$ is the average jump size measured as a proportional increase in the asset price(mean/magnitude of the random jumps); $(q-1)$ is the random variable percentage change in the asset price if the poisson event occurs; ϵ is the expectation operator over the random variable q; σ is the volatility of the diffusion compound; dW_t is the standard Wiener process(Brownian motion); dq_t is the independent poisson process generating the jumps. which can therefore be described as $\frac{dS_t}{S_t} = (\mu - \lambda k) + \sigma dW_t$ if the event does not occur or $\frac{dS_t}{S_t} = (\mu - \lambda\ k) + \sigma\ dW_t + q - 1$ if the poisson event does occur Thus if $\lambda = 0$ and also $q - 1 = 0$, then the stock price return reduces to Black-Scholes model which gives;

$$S_t = S_0 \exp[(\mu - \lambda k - \frac{\sigma^2}{2})t + \sigma W_t] \prod_{i=1}^{i=N} q_i \tag{4}$$

Where $N(t)$ is a poisson process with rate λ, $Z(t)$ is a standard Brownian motion and μ is the drift rate. q_i is a sequence of independent identically distributed (i.i.d) non-negative random variables. Merton (1976) assumed that $log(q_i) = Y_i$ is the absolute asset price jump size and normally distributed.

2.4. Logistic Brownian Motion(Non-Linear Brownian Motion)

We obtain Logistic Brownian motion by introducing excess demand functions in the frame- work of the Walrasian (Walrasian-Samuelson) price adjustment mechanism Onyango [21]. The asset price changes are directly driven by excess demand for a security. This is the core principle of Standard Walrasian model. To simply the work we do not allow cross-security effects that might be experienced when the market is multi-security market for which price of one security reacts to the excess demand of another. The dynamic adjustment rule in such simplified markets may be expressed in continuous-time Walrasian-Samuelson form by a rate of return;

$$\frac{1}{S_t}\frac{d}{dt}S_t = kED[S_t] \tag{5}$$

where the parameters t represents continuous time, and $k > 0$ is a positive market adjustment coefficient (known as speed of market adjustment). $ED(S_t)$ is excess demand taken as continuous function of price S_t. In terms of supply and demand functions $QS(S_t)$ and $QD(S_t)$, the excess demand is given by;

$$ED(S_t) = QD(S_t) - QS(S_t) \tag{6}$$

The Walrasian-Samuelson adjustment mechanisms of the j^{th} asset is given by;

$$\begin{aligned} \frac{1}{S_j(t)}\frac{d}{dt}S_j(t) &= h_j(ED_j(S_t) \quad && \text{if } j = 1, 2, 3, \dots, n \\ \frac{1}{S_j(t)}\frac{d}{dt}S_j(t) &= 0 && \text{if } S_t = 0 \end{aligned}$$

Where $S_j(t)$ is the price of the j^{th} asset, $h_j ED(S_t)$ is the total excess demand function for the j^{th} asset as a function of S_t for the whole market and h_j is any (fixed) monotonic increasing differentiable real-valued function Samuelson [23]. Approximating h_j by linear function, we have

$$\frac{1}{S_j(t)}\frac{d}{dt}S_j(t) = k_j[Q_{D_j}(S(t)) - Q_{S_j(S(t))}] \tag{7}$$

where k_j is a positive adjustment coefficient and interpreted as "the speed of adjustment" of the market changes in supply and demand. In deterministic price adjustment model we can make price adjustment model more computational, we take supply and demand functions to be fixed functions instantaneous price S_t. Then at Walrasian equilibrium price point S^*, $Q_D(S^*) = Q_S(S^*)$, since excess demand is equal to zero. On the other assumption of fixed supply and demand curves, S^* is constant. Away from equilibrium, excess demand for the security will raise its price S_t. And an excess supply will lower its price. Thus the rate of change of price, S_t, with respect to time, t, will depend on the sign of the excess demand. The linear form of $Q_D(S_t)$ and $Q_S(S_t)$ about the constant equilibrium price S_t, gives the deterministic model of price adjustment as;

$$\frac{1}{S_t}\frac{d}{dt}S_t = k(\alpha + \beta)(S^* - S_t) \tag{8}$$

where $Q_D(S_t) = \alpha(S^* - S_t)$, $Q_S(S_t) = -\beta(S^* - S_t)$ and a constant α and β are demand and supply sensitivities respectively. In logistic Brownian motion we model random fluctuations in supply and demand by changes $\delta\alpha$ and $\delta\beta$ in their respective sensitivities. We consider that $Q_D(S_t)$ and $Q_S(S_t)$ to represent average effects of demand and supply, and suppose that both curves steepen or level off in response to random observed trades, cumulatively they execute a random walk or Wiener diffusion process.

From the above models we have

$$\frac{dS_t}{S_t(S^* - S_t)} = k(\alpha + \beta)(\delta\alpha - \delta\beta)dt \tag{9}$$

From equation above with $k(\alpha + \beta) = \mu$ (logistic growth parameter) and $(\delta\alpha - \delta\beta)dt = \sigma dW$ (noise process) we obtain the simpler form

$$\frac{dS_t}{S_t(S^* - S_t)} = \mu dt + \sigma dW \tag{10}$$

Given that trading produces many small random shocks, it is plausible to suppose that $W = \epsilon\sqrt{t}$, with random number ϵ following a standard normal distribution. Parameter σ is analogous to the price volatility of security trading in the steadier market conditions modelled by dynamical geometric Brownian equation. The equation defines an Ito's Process evolving according to the stochastic differential equation;

$$dS_t = \mu S_t(S^* - S_t)dt + \sigma S_t(S^* - S_t)dW \tag{11}$$

We refer to this equation as logistic Brownian motion model or logistic stochastic differential equation. Where S_t is the price of the underlying asset at any time t, S^* is the market equilibrium, μ is the rate of increase of the asset price, σ is the volatility of the underlying asset and $dW_t \sim N(0; dt)$. Using the Heaveside coverup method, the solution of the model is given by;

$$S_t = \frac{S^* S_0}{S_0 + (S^* - S_0)\exp -(\mu S^*(t - t_0) + \sigma S^* W_t)} \tag{12}$$

This price dynamic is referred to us as logistic Brownian motion of stock price S_t Oduor [20].

2.5. Logistic Brownian Motion with jump diffusion

Using the stochastic differential equation and incorporating the jump diffusion process in Geometric Brownian motion we get;

$$\frac{dS_t}{S_t(S^* - S_t)} = (\mu - \lambda k)dt + \sigma dW + dq \tag{13}$$

Solving for S_t using the heaveside coverup method, we will finally get

$$S_t = \frac{S^* S_0}{S_0 + (S^* - S_0)\exp -[(\mu - \lambda k)S^*(t - t_0) + \sigma S^* W_t + q_t S^*]} \tag{14}$$

This price dynamic is referred to us as logistic Brownian motion with jump diffusion of stock price S_t, with the initial price S_0, equilibrium price S^*, μ is the expected return from the asset, λ is the rate at which jumps happen and k is the average jump size measured as a proportional increase in asset price and q is the poison process generating jumps.

3. The Price of Dividend Yielding Asset that Follows a Logistic Brownian Motion With Jump Diffusion Process

Using logistic brownian motion with jump diffusion and incorporating the dividend yielding rate since in reality, assets do pay dividends to the shareholders. In this case, we consider payments of dividends to be continuous.We consider a logistic jump diffusion model given by;

$$\frac{dS_t}{S_t(S^* - S_t)} = (\mu - \lambda k)dt + \sigma dW + dq \tag{15}$$

Incorporating the dividend yielding rate, the resulting equation becomes;

$$\frac{dS_t}{S_t(S^* - S_t)} = (\mu - \gamma - \lambda k)dt + \sigma dW + dq \tag{16}$$

where γ is the dividend yielding rate. The L.H.S of equation (16) can be done using Heaveside coverup method, we obtain;

$$\frac{dS_t}{S_t(S^* - S_t)} = \frac{A}{S_t} + \frac{B}{(S^* - S_t)} \tag{17}$$

To solve for A,

$$\frac{dS_t}{S_t(S^* - S_t)} = \frac{A}{St}$$

this implies that

$$A = \frac{S_t dS_t}{S_t(S^* - S_t)}$$

simplifying and letting $S_t = 0$ we obtain

$$A = \frac{1}{S^*} dS_t \tag{18}$$

To solve for B,

$$\frac{dS_t}{S_t(S^* - S_t)} = \frac{B}{(S^* - S_t)}$$

which implies that

$$B = \frac{(S^* - S_t)dS_t}{S_t(S^* - S_t)}$$

simplifying and letting $S^* = S_t$ we obtain

$$B = \frac{1}{S^*} dS_t \tag{19}$$

Substituting equation (18) and equation (19) into equation (17) we get;

$$\frac{dS_t}{S_t(S^* - S_t)} = \frac{\frac{1}{S^*} dS_t}{S_t} + \frac{\frac{1}{S^*} dS_t}{(S^* - S_t)} \tag{20}$$

which implies that;

$$\frac{\frac{1}{S^*} dS_t}{S_t} + \frac{\frac{1}{S^*} dS_t}{(S^* - S_t)} = (\mu - \gamma - \lambda k)dt + \sigma dW + dq \tag{21}$$

Integrating the equation (21) with respect to t, we obtain;

$$\frac{1}{S^*} \ln |S_t| + \frac{1}{S^*}(- \ln |S^* - S_t|_{t_0}^t) = (\mu - \gamma - \lambda k)t/_{t_0}^t + \sigma W(t) + q(t) \tag{22}$$

We then obtain;

$$\frac{1}{S^*} \ln |\frac{S_t}{S^* - S_t}|_{t_0}^t = (\mu - \gamma - \lambda k)t/_{t_0}^t + \sigma W(t) + q(t) \tag{23}$$

By letting $S_{t_0} = S_0$

$$\ln|\frac{S_t}{S^*-S_t}| - \ln|\frac{S_0}{S^*-S_0}| = (\mu-\gamma-\lambda k)S^*(t-t_0) + \sigma W(t)S^* + q(t)S^* \tag{24}$$

By merging the L.H.S, we obtain;

$$\ln|\frac{S_t(S^*-S_0)}{(S^*-S_t)S_0}| = (\mu-\gamma-\lambda k)S^*(t-t_0) + \sigma W(t)S^* + q(t)S^* \tag{25}$$

Let $(\mu-\gamma-\lambda k)S^*(t-t_0) + \sigma W(t)S^* + q(t)S^* = \tau$

$$\ln|\frac{S_t(S^*-S_0)}{(S^*-S_t)S_0}| = \tau \tag{26}$$

$$\frac{S_t(S^*-S_0)}{(S^*-S_t)S_0} = \exp\tau \tag{27}$$

$$S_t(S^*-S_0)\exp-\tau = (S^*-S_t)S_0 \tag{28}$$

which is similar to;

$$S_t(S^*-S_0)\exp-\tau = S^*S_0 - S_tS_0 \tag{29}$$

$$S_t(S^*-S_0)\exp-\tau + S_tS_0 = S^*S_0 \tag{30}$$

Factorizing S_t in the L.H.S we obtain

$$S_t[(S^*-S_0)\exp-\tau + S_0] = S^*S_0 \tag{31}$$

This implies that;

$$S_t = \frac{S^*S_0}{S_0 + (S^*-S_0)\exp-\tau} \tag{32}$$

Replacing the value of τ we obtain;

$$S_t = \frac{S^*S_0}{S_0 + (S^*-S_0)e^{-[(\mu-\gamma-\lambda k)S^*(t-t_0)+\sigma W(t)S^*+q(t)S^*]}} \tag{33}$$

This price dynamic is referred to us as the price of dividend yielding asset that follows a logistic Brownian motion with jump diffusion process.

Where S_t is the asset price, μ is the asset price growth rate, S^* is the equilibrium price of the asset, S_0 is the initial price of the asset, γ is the dividend yielding rate of the asset, k is the average jump size, λ is the rate at which jumps happen, q is the poisson process generating the jump, σ is the volatility of the market price, $W(t)$ is the Wiener process(Brownian motion).

4. Conclusion

In this paper, we introduced the concept of dividend yielding rate of the asset in the logistic brownian motion with jump diffusion process. The model obtained is useful to the long term investors to know the impact of jump diffusion process of stocks on assets that yield dividends before allocating decision and profitability of trading strategies to invest on high dividend yielding assets. It is also be important as it will help the investors to know whether or not asset returns exhibit jump diffusion.

References

[1] R. Apaka, S. N. Onyango, O. Omolo and O. Nyakinda, *Modelling of Jumps and Mean Reversion in Kenya Interest rates*, International Journal of marketing and Technology, 5(11)(2015), 8-11.

[2] E. Bayraktar and H. Xing, *Pricing Asian Options for jump Diffusion*, The Journal of Mathematical Finance, 49(2010), 1467-9965.

[3] F. Black and N. Scholes, *The pricing options and corporate liabilities*, Journal of political Economy, 18(1973), 637-659.

[4] Chao Li, *Option pricing with generalized continuous time random walk models*, Phd Thesis, Queen Mary University of London, (2016).

[5] J. C. Cox and A. S. Ross, *The Valuation of Options for alternative Stochastic Processes*, Journal of Financial Economics, 3(1976).

[6] David M. Cutler, James M. Poterba and Lawrence H. Summers, *Speculative dynamics*, Review of Economic Studies, 58(1991), 529-546.

[7] Werner F. M. De Bondt and Richard Thaler, *Does the Stock Market Overreact?*, Journal of Finance, 40(1985), 793-805.

[8] Charles Engel and Charles S. Morris, *Challenges to Stock Market Efficiency: Evidence from Mean Reversion Studies*, Economic Review, Federal Reserve Bank of Kansas City, 76(Sep)(1991), 21-35.

[9] Eugene Fama and Kenneth French, *Permanent and Temporary Components of Stock Prices*, Journal of Political Economy, 96(1988), 246-273.

[10] Farouk Tijjani Sa'ad, *Pricing of Compound Options*, Masters Thesis, African University of Science and Technology, Abuja-Nigeria, (2013).

[11] Z. Huang and S. G. Kou, *First passage times and analytical solutions for options on two assets with jump risk*, Preprint, Columbia University, (2006).

[12] C. J. Hull, *Futures and other derivatives. Joseph L.Rotman School of Management*, 4th Edition, University of Toronto, (2000).

[13] J. Hull and A. White, *The pricing of options on assets with stochastic volatility*, Journal of Finance, 42(1987).

[14] S. G. Kou, *A Jump-Diffusion Model for Option Pricing*, Management Science, 48(8)(2002), 1086-1101.

[15] Andrew Lo and Craig A. MacKinlay, *Stosk Market Prices Do Not Follow Random Walks: Evidence From a Single Specification Test*, Review of Financial Studies, 1(1988), 41-66.

[16] Robert C. Merton, *Option Pricing When Underlying Stock Returns are Discontinuous*, Masachusetts Institute of Technology, 16(22)(1975), 225-443.

[17] Otula J. Nyakinda, *Derivation of Non-Linear Logistic Black-Scholes-Merton Partial Differential Equation*, PhD Thesis, Jaramogi Oginga Odinga University of Science and Technology, (2011).

[18] Otula J. Nyakinda, *A Logistic Non-Linear Black-Scholes Partial Differential Equation: European Option*, International Journal of Research-Granthalaayan, 6(6)(2018), 480-487.

[19] D. B. Oduor, O. N. Omolo, N. B. Okelo and S. N. Onyango, *Estimation of Market Volatility: A case of logistic Brownian motion*, International Journal of marketing and Technology, 2(1)(2012), 32-48.

[20] D. B. Oduor and S. N. Onyango, *A logistic Brownian motion with a price of dividend yielding Asset*, International Journal of Research and Social Sciences, 2(2)(2012), 44-52.

[21] S. N. Onyango, *Extracting Stochastic process from market price data: A pattern recognition approach*, PhD Thesis, University of Huddersfield, UK, (2003).

[22] P. Reed, *On the rate of growth of population of United States since 1938 and its mathematical representation*, Proceeds of national academy of science, 6(1920), 275-288.

[23] P. A. Samuelson, *The stability of equilibrium, Comparative Statistics and Dynamics*, Econometrics, 9(1965), 97-120.

[24] P. A. Verlhust, *Notice on the law that the population follows in its increase*, Mathematical and Physical Correspondence, 10(1838), 113-121.

[25] S. H. Wang, *Jump diffusion model*, In: Lee CF., Lee A.C. (eds) Encyclopedia of Finance, Springer, Boston, MA, (2006).

[26] P. Wilmott, S. Howlson and J. Dewayne, *The Mathematics of Financial Derivatives*, Pass Syndicate of The University of Cambridge, (1995).