JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR 2ND SEMESTER 2021/2022 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE:WAB 2310

COURSE TITLE: TEST OF HYPOTHESES

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a. Describe the following concepts of hypothesis tests
   i. Type II error
   ii. The likelihood ratio test
   iii. A test statistic
   iv. Power of a statistical test
   v. Critical value
   vi. P-value **[6 marks]**

b. A random sample of 100 observations from a quantitative population produced a sample mean of 26.8 and a standard deviation of 6.5.
   i. Use the p-value approach to determine whether the mean is different from 28. **[4 marks]**
   ii. What is the power of the test if in fact the mean is 27.6 **[4 marks]**

c. The following $n = 10$ observations are a sample from a normal population;
   7.4, 7.1, 6.5, 7.5, 7.6, 6.3, 6.9, 7.7, 6.5, 7.0
   Test H_0: $\mu = 7.5\ against\ H_1$: $\mu < 7.5.\ Use\ \alpha =\ 0.01$ **[5 marks]**

d. Let $X_1, X_2, \dots, X_k$ be a random sample from the binomial distribution:$Bin(n, p)$ Find the LR test of level α for testing $H_0 :\ p\ =\ 0.4$ versus $H_1 :\ p = 0.6$.
   [6 marks]

e. A precision instrument is guaranteed to read accurately to within two units. A sample of four instrument readings on the same object yielded the measurements:
   353, 351, 351 and 355. Does the data provide sufficient evidence to show that $\sigma^2 > 3, test\ at\ \alpha = 0.05$ **[5 marks]**

QUESTION TWO (20 MARKS)

a. Eight individuals were put on two different stimuli to test their reaction times to a command. The reaction times in seconds due to the two different stimuli were recorded as follows;

Stimulus I	3	1	1	2	1	2	3	2
Stimulus II	4	2	3	1	2	3	3	3

i. State any two assumptions, for the use of t test in this situation. **[2 marks]**

ii. Does the data provide enough evidence to indicate a difference in mean reaction times for the two stimuli? Test at $\alpha = 0.05$ **[6 marks]**

iii. Find the approximate $p - value$ for the test. Does this value confirm your conclusions? **[4 marks]**

b. Let $y_1, y_2, \ldots, y_{10}$ be a random sample from the $N(\theta, 1)$ where $\theta = 5$ or $\theta = 6$. Using Neyman –Pearson Lemma, find the best test for testing $H_0: \theta = 6$ $versus$ $H_1: \theta = 5$ at 5% significance level. **[8.marks]**

QUESTION THREE (20 MARKS)

a. Some research was carried out to test lead levels in water consumed by residents in two sections of a city. 100 samples were taken from each of the sections and the following means and standard deviations recorded.

	Section A	Section B
Mean	34.1	36.0
Standard deviation	5.9	6.0

i. State an appropriate Null and alternative hypothesis in testing for difference of means for the lead levels. **[1 mark]**

ii. Calculate the test statistic and its p- value to test for a difference in the two population means. Use the p-value to evaluate the statistical significance of the results at 5% level**.** **[6 marks]**

iii. Use a 95% confidence interval to estimate the difference in mean lead levels for the two sections. Make a comment on the outcome? **[5 marks]**

b. Let $X_1, X_2, \ldots, X_n$ be a random sample from the Normal distribution: $N(\theta, 36)$. Find a uniformly most powerful test critical region of size $\alpha = 0.05$ for testing $H_o: \theta = 18$ against $H_1: \theta < 18$ **[8 marks]**

QUESTION FOUR (20 MARKS)

a. A single observation is taken from a Poisson distribution with mean θ and used to test the hypothesis $\theta = 4.5$ against the alternative$\theta > 4.5$. The critical region is chosen to be $x \geq 11$.

i. At what significance level is the test carried out? **[5 marks]**

ii. Find the power of the test if in fact $\theta = 6.5$ **[5 marks]**

b. Test at 5% level of significance whether or not the following samples have come from the same normal population. **[10 marks]**

Sample A	127	195	162	170	143	205	168	175	197	136	
Sample B	135	200	160	182	147	200	172	186	194	141	155

QUESTION FIVE (20 MARKS)

a. A bearing used in an automotive application is supposed to have an inside diameter of 3.81cm. A random sample of 25 bearings is selected and the average inside diameter of these bearings is 3.8037cm. Bearing diameter is known to be normally distributed with standard deviation 0.03cm.

i. Test the hypothesis that the mean is different from what is known. **[5 marks]**

ii. What sample size would be required to detect a true mean diameter as low as 3.797cm if we wanted the power to be at least 0.9? **[5 marks]**

b. A random variable can be modeled by a binomial distribution with parameters n = 10 and P whose value is unknown. Find the critical region for test of:
$H_O : P = 0.5 \; against \; H_1 : P \neq 0.5$ at;

i. 5% level of significance **[5 marks]**

ii. 10% level of significance **[5 marks]**