JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR 2ND SEMESTER 2021/2022

REGULAR (MAIN)

COURSE CODE: WAB 2301

COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATIONS I

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

a. Find the stochastic differential equation for W_t^2. (2marks)

b. Explain what is meant by the continuous-time lognormal model of security prices.

(4marks)

c. When valuing derivatives it is often assumed that the price of the underlying security follows a geometric Brownian motion with stochastic differential equation:
$dS_t = S_t(\mu dt + \sigma dZ_t)$
where Z_t represents a standard Brownian motion. List the 3 advantages and 3 disadvantages of this assumption. (6marks)

d. i.) Explain what it means for a bond to be default-free. (2marks)
ii.) State four possible outcomes of a default. (2marks)
iii.) List four types of credit event. (4marks)
iv.) Explain what is meant by the recovery rate for a bond. (2marks)

e. Explain what is meant by:
i.) Structural models . (2marks)
ii.) Reduced-form models. (2marks)
iii.) Intensity-based models. (2marks)

f. Explain what is meant by a credit spread. (2marks)

QUESTION TWO

a. A process X_t satisfies the stochastic differential equation:
$dX_t = \sigma(X_t)dB + \mu(X_t)dt$
where B_t is a standard Brownian motion.
Deduce the stochastic differential equation for the process X_t^3. (10marks)

b. Find the stochastic differential equation for B_t^2. (5marks)

c. The shares of Abingdon Life can be modelled using a lognormal model in which the drift parameter, $\mu=0.104$ pa and volatility, $\sigma=0.40$ pa. If the current share price is 2.00, derive a 95% confidence interval for the share price in one week's time, assuming that there are exactly 52 weeks in a year. (5marks)

QUESTION THREE

a. Briefly explain the following (10 marks)
   i) Yield Curve
   ii) Inverted yield curve
   iii) Spot Rate
   iv) Immunization
   v) Forward Rate

b. Bond A is a 10-year bond with 10% coupon. Its price is P_A = 98.72. Bond B is a 10-year bond with an 8% coupon. Its price is P_B = 85.89. Both bond have the same face value, normalized to 100. Determine S_{10} . (5 marks)

c. If the spot rates for 1 and 2 years are s_1 =6.3% and s_2 = 6.9%, what is the forward rate $f_{1,2}$? (5 marks)

QUESTION FOUR

a. i.) Write down Ito's Lemma as it applies to a function $f(X_t)$ of a stochastic process X_t that satisfies the stochastic differential equation
   $dX_t = \sigma_t dB_t + \mu_t dt$, where B_t is a standard Brownian motion. (2marks)

   ii.) Hence find the stochastic differential equations for each of the following processes:
   (a) $G_t = \exp(X_t)$
   (b) $Q_t = X_t^2$
   (c) $V_t = (1+ X_t)^{-1}$
   (d) $L_t= 100 + 10X_t$
   (e) $J_t = \ln B_t$
   (f) $K_t = 5B_t^3 + 2B_t$ (18 marks)

QUESTION FIVE

a. Company X has just issued some 5-year zero-coupon bonds. A continuous-time two-state model is to be used to model the status of the company and to calculate the fair price of the bonds. It is believed that the risk-neutral transition rate for failure of the company is $\lambda(t)=0.002t$, where t is the time in years since the issue of the bonds. The 5-year risk-free spot yield is 5.25% expressed as an annual effective rate.

(i) Calculate the risk-neutral probability that the company will have failed by the end of 5 years. (5marks)

(ii) In the event of failure of the company, the bonds will make a reduced payment at the maturity date. The recovery rate for a payment due at time t is: $\delta(t)=1-0.05t$

Calculate the fair price to pay for £100 nominal of a Company X bond, taking into account the possibility of company failure. (5marks)

b. Let $\{X_t\}$ be a continuous-time stochastic process defined by the equation $X_t= \alpha W_t^2+ \beta$,

Where $\{W_t\}$ is a standard Brownian motion and αandβ are constants.

By applying Ito's Lemma, or otherwise, write down the stochastic differential equation satisfied by X_t. (10marks)