



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION
AND ACTUARIAL SCIENCE**

3RD YEAR 1ST SEMESTER 2022/2023 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: 2302

COURSE TITLE: FINANCIAL MATHEMATICS II

EXAM VENUE:

STREAM: EDUCATION, ACTUARIAL

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question one (compulsory) and any other two questions.**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1 [30 marks]

- a. Define the moments of S_n and show that the variance is given by

$$\text{Var}[S_n] = [(1+j)^2 + S^2]^n - (1+j)^{2n} \quad [8\text{marks}]$$

- b. With well-illustrated graphs, explain the three theories of a yield curves.

[6marks]

- c. A payment of £800 is due in 5 years' time. Calculate the present value of this payment at a force of interest of 9% *pa*.

[3marks]

- d. Suppose a stock has two assets A and B with the following returns: Asset A=5,7,3,12,15,17,8,1,14,10 and asset B=20,15,2,11,7,8,12,3,4,9. Suppose we have the correlation coefficient of 0.1 and weighted average of return of A = 0.7. Calculate the mean and the variance portfolio of stock.

[5marks]

- e. Calculate the rate of return of the following portfolio of the three assets

[4marks]

| Security | Number of shares | price | rate of return | |
|----------|------------------|-------|----------------|--|
| A | 250 | 50 | 20% | |
| B | 500 | 30 | 17% | |
| C | 300 | 20 | 27% | |

- f. A fixed-interest security with a 6% annual coupon payable half-yearly in arrears is purchased at a price that gives a gross effective yield of 10% *pa* by an investor who is subject to capital gains tax at 30%. It is redeemable at par after 15 years. Calculate the amount of capital gains tax payable per £100 nominal.

[4marks]

Question 2 [20 marks]

- a. Define the accumulation factor $A(t, t+h)$ and give a formula for the force of interest $\delta(t)$ per unit time in terms of the accumulation factor.

[4marks]

- b. The force of interest $\delta(t)$ at time t (measured in years) is given by $\delta(t) = 0.01t + 0.04$.

- i. Calculate the corresponding nominal rate of interest for the period $t = 1$ to $t = 2$.

[6marks]

- ii. If an investment of 1 is made at time $t = 1/2$, calculate the value to which it will have accumulated by time $t = 6$.

[4marks]

- c. Calculate the accumulated value after 6 months of an investment of £100 at time 0 at the following rates of interest:

- i. a force of interest of 0.05 *pa*

[2marks]

- ii. a rate of interest of 5% *pa* convertible monthly

[2marks]

- iii. an effective rate of interest of 5% *pa*.

[2marks]

Question 3 [20 marks]

a. From the given table below

| Asset A | Asset B | Market |
|---------|---------|--------|
| 20 | 5 | 10 |
| 11 | -3 | 12 |
| -4 | 5 | 8 |
| 9 | 13 | 5 |
| 5 | 2 | 8 |

$R_f = 5\%$

Calculate

[12marks]

- The beta of each asset
- The Sharpe measure for each asset and rank of the assets
- The Treynor measure of each of the asset and rank them

b. An investor has the choice of the following assets that earn rates of return as follows in each of the four possible states of the world:

| State | Probability | Asset 1 | Asset 2 | Asset 3 |
|-------|-------------|---------|---------|---------|
| 1 | 0.2 | 5% | 5% | 6% |
| 2 | 0.3 | 5% | 12% | 5% |
| 3 | 0.1 | 5% | 3% | 4% |
| 4 | 0.4 | 5% | 1% | 7% |

Market capitalization 10,000 17,546 82,454

Determine the market price of risk assuming CAPM holds.

[8marks]

Question 4 [20 Marks]

a. At 1 July 2004, an investor has a liability of £20,000 to be paid on 1 January 2008 and a liability of £18,000 to be paid on 1 July 2010. The investor currently holds assets with a present value equal to the present value of the liabilities. The investor wishes to immunise his position by investing in two zero coupon bonds with outstanding terms of four years and seven years. Determine whether or not this is possible assuming an effective interest rate of 10% per annum.

[8marks]

b. A fund must make payments of £50,000 at the end of the sixth and eighth years. Show that, if interest rates are currently 15% *pa* at all durations, immunisation to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5- year zero-coupon bond and a 10-year zero-coupon bond.

[12marks]

Question 5 [20 Marks]

a. An investor is contemplating an investment with a return of £ R , where:

$$R = 300,000 - 500,000U$$

where U is a uniform $[0,1]$ random variable.

Calculate each of the following four measures of risk:

[12marks]

- variance of return
- downside semi-variance of return
- shortfall probability, where the shortfall level is £100,000
- Value at Risk at the 5% level.

- b. Calculate the mean and variance of the accumulated value of an initial investment of £40,000 at the end of 25 years if the annual rates of return are assumed to conform to the varying interest rate model and follow a *Gamma* (16,200) distribution.
[8marks]