JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE, EDUCATIONARTS AND SPECIAL EDUCATION

2ND YEAR 1ST SEMESTER 2021/2022 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WMB 9203

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Let X and Y have joint density function given by

$$f(x,y)=\begin{cases} c(x+3y) & 0<x<2 \quad 0<y<2 \\ 0 & otherwise \end{cases},$$

i. Calculate the value of c. (3 Marks)

ii. Hence calculate $\Pr(x<1, y>0.5)$ (3 Marks)

b) Given that M and N are random variables with joint probability function given by;

$$P(m,n)=\begin{cases} \dfrac{m}{35\times 2^{n-2}} & m=1,2,3,4 \quad n=1,2,3 \\ 0 & otherwise \end{cases}$$

i. Obtain the marginal distribution of N. (3 Marks)

ii. Find the conditional probability function of M given $N=n$. (2 Marks)

c) If X and Y are random variables with joint probability distribution function given by $f(x,y)=\begin{cases} e^{-x-y} & x>0 \quad y>0 \\ 0 & otherwise \end{cases}$, determine whether or not X and Y are independent random variables. (5 Marks)

d) Verify that $E[X^2+2Y]=E[X^2]+E[2Y]$ for the random variables X and Y with the joint probability distribution functions given in the table below (7 Marks)

		X		
		0	1	2
Y	1	0.1	0.1	0
	2	0.1	0.1	0.2
	3	0.2	0.1	0.1

e) Show that by simplification $\operatorname{cov}(X,Y)=E(XY)-E(X)E(Y)$ (3 Marks)

f) Suppose X and Y are random variables with mean of $X=20$, mean of $Y=4$, $\operatorname{var}(X)=9$, $\operatorname{var}(Y)=4$ and coefficient of correlation is $-\frac{1}{6}$. Determine the mean of $(X-3Y+4)$ and $\operatorname{var}(X-3Y+4)$ (4 Marks)

QUESTION TWO (20 MARKS)

Obtain the coefficient of correlation between two random variables X and Y with joint probability distribution function given by $f(x,y)=\begin{cases} \frac{1}{3}(x+y) & 0<x<1 \quad 0<y<2 \\ 0 & otherwise \end{cases}$

QUESTION THREE (20 MARKS)

a) The continuous random variables U and V have joint probability density functions given by $f_{U,V}(u,v)=\begin{cases} \dfrac{2u+v}{3000} & 10<u<20 \quad -5<v<5 \\ 0 & otherwise \end{cases}$

i. Find $\Pr[10<u<15, v>0]$ (4 Marks)

ii. Obtain the mean of U and mean of V. (6 Marks)

iii. Determine the marginal probability density function for U and V (4 Marks)

b) A bivariate distribution has the following probability function given by the table below

		X		
		1	2	3
Y	1	0.10	0.10	0.05
	2	0.15	0.10	0.05
	3	0.20	0.05	0
	4	0.15	0.05	0

Determine P(X=1), P(X=2) and P(X=3) (6 Marks)

QUESTION FOUR (20 MARKS)

The joint probability density of X and Y is given as

$$f(x,y)=\begin{cases}3(x+y) & 0<x+y<1 \quad 0<y<1 \quad 0<x<1\\ 0 & otherwise\end{cases}$$

a) Find the marginal density of X (3 Marks)

b) Find $\Pr[X+Y<0.5]$ (4 Marks)

c) Find $E[Y \mid X=x]$ hence obtain $E[Y \mid X=0.5]$ (6 Marks)

d) Find $E[X^2 \mid Y=0.5]$ (7 Marks)

QUESTION FIVE (20 MARKS)

a) Obtain the mean and variance of a gamma distribution with parameters r and λ given by the following probability distribution function

$$f_X(x)=\begin{cases}\frac{\lambda}{\Gamma(r)}(\lambda x)^{r-1}e^{-\lambda x} & r,\lambda>0 \quad x\geq 0\\ 0 & otherwise\end{cases}$$ (10 Marks)

b) Write down the probability distribution function of a bivariate normal distribution with parameters

i. $\mu_x=0$, $\mu_y=4$, $\sigma_x^2=1$, $\sigma_y^2=9$ and $\rho=2/3$ (3 Marks)

ii. $\mu_x=1$, $\mu_y=2$, $\sigma_x^2=1$, $\sigma_y^2=6$ and $\rho=0$ (3 Marks)

c) Compute the $\operatorname{cov}(X,Y)$ from the table below that shows the joint probability distribution function of discrete random variables X and Y given by the table below (4 Marks)

		Y		
		0	1	2
X	0	0.10	0.10	0.20
	1	0	0.15	0.05
	3	0.10	0.20	0.10