Abstract

The study of elementary operators has been of great interest dating back to many decades when it was initiated by Sylvester in 1884. A number of scholars have described several properties of elementary operators arising from their structure. In all these studies, the authors considered general elementary operators but not the normally represented elementary operators. In this study, we considered the normally represented elementary operators. Our objectives for the study entailed establishment of normality conditions of operators in Hilbert space inducing the elementary operators and describing the norms of normally represented elementary operators. To do this, we needed some fundamental principles, known inequalities e.g polarization identity, parallelogram law and Cauchy- Buniakowsky -Schwarz inequality. We also employed some technical approaches like tensor products and orthogonal direct sum. Some of our results show that for an operator A to be normal, it is necessary that A is self adjoint. It is also sufficient that $AA^* = A^*A$ makes an operator to be normal. Further results show that the norm of an elementary operator is equal to the largest singular value of the operator itself i.e $S_i(\mathcal{M}) = ||\mathcal{M}||$ and also if $\mathcal{U}_{A,B} = A \otimes_h B + B \otimes_h A$ is normally represented, then $||\mathcal{U}_{A,B}||_{Inj} \geq 2(\sqrt{2}-1)||A||||B||$. The results of this study are useful in the fields of quantum chemistry and physics in generation of quantum bits and estimation of ground state energies of chemical systems and subsystems.