Abstract

Advection-diffusion equation arises from modeling of physical processes in a wide range of scientific disciplines. The (3+1) dimensional Advection-Diffusion equation we derived can generally be written as; $C_t = f_1(x,y,z,t)C_{xx}+f_2(x,y,z,t)C_{yy}+f_3(x,y,z,t)C_{zz}+f_4(x,y,z,t)C_{xy}+f_5(x,y,z,t)C_x+f_6(x,y,z,t)C_y$. The advection terms C_x and C_y describe how water is carried along an underground layer of water-bearing permeable rock called aquifer due to bulk fluid motion. The diffusion terms C_{xx}, C_{yy}, C_{zz} and C_{xy} describe the spreading of contaminant due to random molecular collision and C_t defines the rate of change of concentration with respect to time. (1+1) ADE was derived and solved to investigate water quality but the results could not be put to comprehensive physical interpretation in the absence of a lateral and a vertical dimension. (2+1) ADE was also derived and solved to investigate contaminant concentration in aquifer but the results again could not be subjected to physical interpretation because the solution was in series form,the advection and diffusion parameters were arbitrarily assigned and the effect of diffusion on the vertical plane could not be ascertained. It was therefore important to derive and solve the (3+1) ADE with a mixed derivative to investigate the effect of varying exponential and decaying Advection-Diffusion parameters on contaminant concentration in aquifer. The model equation that governs the change of concentration with respect to time is solved numerically by Finite Difference Method using $MATLAB$ computer software programme. An Alternating Direction Explicit and Alternating Direction Implicit numerical schemes for the equations are developed and the concepts of Consistency and Stability discussed and analysed. The study found out that both numerical schemes are Consistent with the model equation, implying that the model equation could be recovered from the set of algebraic equations of the schemes developed. Von Neuman method is used to analyse the Stability of the numerical schemes developed and the two schemes are found to be unconditionally stable. The solution for the model equation indicate that contaminant concentration increases with respect to time in both schemes when the diffusion parameters are exponential and the advection parameters are decaying. On the other hand, contaminant concentration decreases with respect to time in both schemes when the diffusion parameter is decaying and the advection parameter is exponential. This study is a big contribution to mathematical knowledge to the extent that the results obtained will guide in the identification of suitable underground water sources drilling sites,complement the current Vertical Electrical Sounding technology which addresses the question of quantity and not quality of underground water and to reduce considerably the cost of water prospecting.