JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2ND YEAR 2ND SEMESTER 2022/2023

REGULAR (MAIN)

COURSE CODE: WAB 2206

COURSE TITLE: ACTUARIAL MATHEMATICS I

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a. Define the following terms (6 marks)
   i.) An annuity certain
   ii.) An "interest-only" loan
   iii.) A mortgage or repayment loan

b. If μ_x takes the constant value 0.001 between ages 25 and 35, calculate the probability that a life aged exactly 25 will survive to age 35. (2marks)

c. Given that $e_{50}=30$ and $\mu_{50-t}=0.005$ for $0 \leq t \leq 1$, what is the value of e_{51}? (3marks)

d. Describe the cashflows for an organisation that issues a zero-coupon bond. (2marks)

e. An investor purchased a three-year index-linked security on 1.1.2001. In return the investor received payments at the end of each year plus a final redemption amount, all of which were increased in line with the index given in the table above. The payments would have been £600 each year and £11,000 on redemption if there had been no inflation. Calculate the payments actually received by the investor. (3marks)

f. An 8-month loan is repayable by a single payment of £100,000. If the loan is issued at a rate of commercial discount of 15% pa, how much was initially lent to the borrower?

(2marks)

g. Find the effective annual interest rate that is equivalent to a simple interest rate of 3% pa over 4 years. (3marks)

h. Define the effective rate of interest over a given time period (2marks)

i. Calculate the present value on 1 September 2002 of payments of £280 due on 1 September 2004 and £360 due on 1 March 2005. Interest is 15% pa effective. (3marks)

j. An investment of £1,000 made at time 0 is accumulated at the following rates: 8% per annum simple for two years, followed by a rate of discount of 6% per annum convertible monthly for two years. Calculate the accumulated amount of the investment after 4 years. (2marks)

k. Find P, if I =5, R =125, i =10% and n = 20. (2marks)

QUESTION TWO (20 MARKS)

a. Show algebraically that $e_x = P_x(1+e_{x+1})$ (5marks)

b. Show that, if mortality experience conforms to Gompertz' Law, then:

$-log(-log\ P_x) = log\left[\frac{\log c}{B(C-1)}\right] - xlogc$

Suggest how this property could be used. (10marks)

c. Show that $Sx(t) = \frac{S(x+t)}{S(x)}$ (5marks)

QUESTION THREE (20 MARKS)

a. The force of interest is given by

$$\delta(t)=\begin{cases}0.08-0.001t & 0 \le t < 3\\ 0.025t-0.04 & 3 \le t < 5\\ 0.03 & 5 \le t\end{cases}$$

Calculate the present value at time 2 of a payment of £1,000 at time 10. (5 marks)

b. If the force of interest is:

$$\delta(t)=\begin{cases}0.08 & 0 \le t < 5\\ 0.13-0.01t & 5 \le t\end{cases}$$

find expressions for the accumulation factor from time 0 to time t. (5marks)

c Derive the following expressions

i. $f_x(t) = -\frac{d}{dt}\,{}_tp_x$

ii. ${}_tq_x = tq_x$ *(if deaths are uniformly distributed between the ages of x and x +1)*

(10 marks)

QUESTION FOUR (20 MARKS)

a. Show that the effective rate of interest, when accumulating using a constant simple interest rate, decreases over time. (5marks)

b. The force of interest is:

$\delta(t)=0.01t + 0.04 \quad 0 \leq t \leq 5$

Find the present value at time 0 of the payment stream $0.5t + 2$, which is received between time 0 and 5. (10marks)

c. An investor deposits £2,000, then withdraws level annual payments starting one year after the deposit was made. Immediately after the 11th annual drawing, the investor has £400 left in the account. Calculate the amount of each withdrawal, given that the annual rate of interest is 8% (5marks)

QUESTION FIVE (20 MARKS)

Show that $fx(t) = {}_tP_x u_{x+t}$ (20 marks)