JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2ND YEAR 2ND SEMESTER 2022/2023

REGULAR (MAIN)

COURSE CODE: WAB 2208

COURSE TITLE: RISK THEORY

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: 19/12/2022 **EXAM SESSION: 15.00-17.00PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [30 MARKS]

a. State the two conditions that must hold for a risk to be insurable. (2 marks)

b. List five other risk criteria that would be considered desirable by a general insurer.

(5 marks)

c. If X has a Pareto distribution with parameters $\lambda = 500$ and $\alpha = 4$, and N has a *Poisson*(50) distribution, calculate the expected value of S . (4 marks)

d. If N has Type 2 negative binomial distribution with $k = 2$ and $p = 0.9$, and X has agamma distribution with $\alpha = 10$ and $\lambda = 0.1$,

   i. Determine an expression for $M_S(t)$. (3 marks)

   ii. Calculate the mean and variance of S. (4 marks)

e. The probability of a claim arising on any given policy in a portfolio of 1,000 one-year termassurance policies is 0.004. Claim amounts have a Gamma(5, 0.002) distribution. Calculate themean and variance of the aggregate claim amount. (4 marks)

f. Suppose that the Poisson parameters of policies in a portfolio are not known but are equally likely to be 0.4 or 0.2.

   i. Find the mean and variance (in terms of $m1$(1^{st} moment) and $m2$(2^{nd} moment)) of the aggregate claims from a policy chosen at random from the portfolio.

   (4 marks)

   ii. Find the mean and variance (in terms of $m1$, $m2$ and n) of the aggregate claims from the whole portfolio. (4 marks)

QUESTION 2 [20 MARKS]

a. A group of policies can give rise to at most two claims in a year. The probability function for the number of claims is as follows:

Number of claims, n	0	1	2
$P(N = n)$	0.7	0.2	0.1

Each claim is either for an amount of 1 or an amount of 2, with equal probability. Claim amounts are independent of one another and are independent of the number of claims. Determine the distribution function of the aggregate annual claim amount, *S.*

(6 marks)

b. A compound random variable $S = X_1 + X_2 + ... + X_N$ has claim number distribution:

$$P(N=n) = 9(n+1)4^{-n-2} \quad , \quad n=0, 1, 2, ...$$

The individual claim size random variable, *X,* is exponentially distributed with mean 2.Calculate $E(S)$ and var(S). (8 marks)

c. The annual aggregate claims from a risk have a compound Poisson distribution with parameter 250. Individual claim amounts have a Pareto distribution with parameters $\alpha = 4$ and $\lambda = 900$. The insurer effects proportional reinsurance with a retained proportion of 75%.Calculate the variances of the total amounts paid by the insurer and by the reinsurer. (6 marks)

QUESTION 3 [20 MARKS]

a. Suppose that K isPoisson (λ), M isPoisson (μ), and K and M are independent. Use a convolution approach to derive the probability function of K+M.(6 marks)

b. The annual aggregate claim amount from a risk has a compound Poisson distribution with Poisson parameter 100. Individual claim amounts are uniformly distributed on (0,20000) . The insurer of this risk has effected excess of loss reinsurance with retention level 16,000. Calculate the mean, variance and coefficient of skewness of both the insurer's andreinsurer's aggregate claims under this reinsurance arrangement.

(8 marks)

c. The annual number of claims from a small group of policies has a Poisson distribution with a mean of 2. Individual claim amounts have the following distribution:

Amount	200	400
Probability	0.4	0.6

Individual claim amounts are independent of each other and are also independent of the number of claims. The insurer has purchased aggregate excess of loss reinsurance with a retention limit of 600. Calculate the probability that the reinsurer is involved in paying the claims that arise in the next policy year. (6 marks)

QUESTION 4 [20 MARKS]

a. Determine an expression for the Moment Generating Function of the aggregate claim amount random variable if the number of claims has a *Bin*(100, 0.01) distribution and individual claim sizes have a *Gamma*(10, 0.2) distribution. (6 marks)

b. The aggregate claims from a risk have a compound Poisson distribution with parameter μ.Individual claim amounts (in £) have a Pareto distribution with parameters α =4 and λ =1,000. The insurer of this risk calculates the premium using a premium loading factor of 0.3 (ie it charges 30% in excess of the risk premium). The insurer is considering effecting individual excess of loss reinsurance with retention limit £1,000. The reinsurance premium would be calculated using a premium loading factor of 0.4. The insurer's profit is defined to be the premium charged by the insurer less the reinsurance premium and less the claims paid by the insurer, net of reinsurance.

   i. Show that the insurer's expected profit before reinsurance is 100μ .(4 marks)

   ii. Calculate the insurer's expected profit after effecting the reinsurance, and hence find the percentage reduction in the insurer's expected profit. (6 marks)

   iii. Calculate the percentage reduction in the standard deviation of the insurer's profit as a result of effecting the reinsurance. (4 marks)

QUESTION 5 [20 MARKS]

a. The distribution of the number of claims from a motor portfolio is negative binomial with Parameters k= 5,000 and $p = 0.9$. The claim size distribution is Pareto with parameters $\alpha = 8$ and λ =1,800. Calculate the mean and standard deviation of the aggregate claim distribution.

(6 marks)

b. Each year an insurance company issues a number of household contents insurance policies, for each of which the annual premium is £80. The aggregate annual claims from a single policy have a compound Poisson distribution; the Poisson parameter is 0.4 and individual claim amounts have a gamma distribution with parameters α and λ . The expense involved in settling a claim is a random variable uniformly distributed between £50 and £ b (>£50). The amount of the expense is independent of the amount of the associated claim. The random variable S represents the total aggregate claims and expenses in one year from this portfolio. It may be assumed that S has approximately a normal distribution.

i. Suppose that:

$\alpha = 1$; $\lambda = 0.01$; b = 100

Show that the company must sell at least 884 policies in a year to be at least 99% sure that the premium income will exceed the claims and expenses outgo.

(7 marks)

ii. Now suppose that the values of α , λ and b are not known with certainty but couldbe anywhere in the following ranges:

$$0.95 \leq \alpha \leq 1.05;\ 0.009 \leq \lambda \leq 0.011;\ 90 \leq b \leq 110$$

By considering what, for the insurance company, would be the worst possible combination of values for α , λ and b , calculate the number of policies the company must sell to be at least 99% sure that the premium income will exceed the claims and expenses outgo.

(7 marks)