JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

3rd YEAR 1ST SEMESTER 2022/2023 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2301

COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATION I

EXAM VENUE: **STREAM: EDUCATION, ACTUARIAL**

DATE: 19/12/2022   **EXAM SESSION: 15.00-17.00PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1 [30 marks]

   a. Define the following terms   [3marks]

i)   Spot rate

ii)   Convexity

iii)   Discrete time forward rate

   b. A fixed interest stock is redeemable at 106% in 15 years' time and pays interest at 9% pa payable half- yearly in arrears. What price should an investor pay to obtain a gross redemption yield of 9% pa?   [4marks]

   c. A fixed-interest security with a 6% annual coupon payable half-yearly in arrears is purchased at a price that gives a gross effective yield of 10% *pa* by an investor who is subject to capital gains tax at 30%. It is redeemable at par after 15 years. Calculate the amount of the capital gains tax payable per £100 nominal.   [4marks]

   d. A government bond pays a coupon half-yearly in arrears of £10 per annum. It is to be redeemed at par in exactly ten years. The gross redemption yield from the bond is 6% per annum convertible half-yearly. Calculate the duration of the bond in years.   [8marks]

   e. Give an expression for the DMT of a level 20-year annuity of 1 *pa* payable annually in arrears and sketch a graph of the Discounted Mean Term in this case as a function of the interest rate. Take the interest rates to be 7%, 8%, 11%, 16% and 20%.   [6marks]

   f. How will the price of conventional gilt that is redeemable at par with an annual coupon of 3% be affected if future rates of interest over all terms increase from 7% to 8%, and the term of the gilt is (a) 5 years and (b) 25 years? Comment on your results.   [5marks]

Question 2 [20 marks]

    i.   State Redington's conditions for immunization   [3marks]

    ii.   Company Z needs to make payments of £60,000 at the end of the 8th and 10th years. Show that, if interest rates are currently 5% *pa* at all durations, immunisation to small changes in interest rates can be achieved by holding an appropriately chosen combination of an 8- year zero-coupon bond and a 15-year zero-coupon bond. [17marks]

Question 3[20 marks]

By 15-05- 2007 the Kenyan government issued an index-linked bond of term 15 years. Coupons are payable half-yearly in arrears, and the annual nominal coupon rate is 4%.

Interest and capital repayments are indexed by reference to the value of a retail price index with a time lag of 8 months. The retail price index value in September 2006 was 200 and in March 2007 were 206.

The issue price of the bond was such that, if the retail price index were to increase continuously at a rate of 7% *pa* from March 2007, a tax-exempt purchaser of the bond at the issue date would obtain a real yield of 3% *pa* convertible half-yearly.

a. Derive the formula for the price of the bond at issue to a tax-exempt investor. [2marks]

b. Show that the issue price of the bond is £111.53%. [10marks]

c. An investor purchases a bond at the price calculated in (i) and holds it to redemption. The retail price index increases continuously at 5% *pa* from March 2007. A new tax is introduced such that the investor pays tax at 40% on any real capital gain, where the real capital gain is the difference between the redemption money and the purchase price revalued according to the retail price index to the redemption date. Tax is only due if the real capital gain is positive.Calculate the real annual yield convertible half-yearly actually obtained by the investor. [8marks]

Question 4[20 marks]

An investor purchased a bond with exactly 15 years to redemption. The bond, redeemable at par, has a gross redemption yield of 5% per annum effective. It pays coupons of 4% per annum, half yearly in arrears. The investor pays tax at 25% on the coupons only.

    a.   Calculate the price paid for the bond. [4marks]

    b.   After exactly eight years, immediately after the payment of the coupon then due, this investor sells the bond to another investor who pays income tax at a rate of 25% and capital gains tax at a rate of 40%. The bond is purchased by the second investor to provide a net return of 6% per annum effective.

    i.   Calculate the price paid by the second investor. [8marks]

    ii.   Calculate, to one decimal place, the annual effective rate of return earned by the first investor during the period for which the bond was held. [8 marks]

Question 5[20 marks]

    a.   An insurance company has liabilities of £10 million due in 10 years' time and £20 million due in 15 years' time, and assets consisting of two zero-coupon bonds, one paying £7.404 million in 2 years' time and the other paying £31.834 million in 25 years' time. The current interest rate is 7% per annum effective.

        (i)   Show that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this insurance company. [8marks]

        (ii)   Determine the profit or loss, expressed as a present value, that the insurance company will make if the interest rate increases immediately to 7.5% per annum effective. [2marks]

        (iii)   Explain how you might have anticipated, before making the calculation in (ii), whether the result would be a profit or loss. [2 marks]

    b.   A government bond pays a coupon half-yearly in arrears of £10 per annum. It is to be redeemed at par in exactly ten years. The gross redemption yield from the bond is 6% per annum convertible half-yearly.

    i.   Calculate the duration of the bond in years. [6marks]

    ii.   Explain why the duration of the bond would be longer if the coupon rate were £8 per annum instead of £10 per annum. [2marks]