JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

2022/2023 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2311

COURSE TITLE: STOCHASTIC PROCESSES 1

EXAM VENUE: **STREAM: ACTUARIAL SCIENCE**

DATE: 15/12/22 **EXAM SESSION: 9.00-11.00AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One Compulsory (30mks)

a) Briefly define the following terms as used in Stochastic Processes (12mks)

   i) Bernouli process
   ii) Trajectory
   iii) Time space
   iv) Markov Property
   v) Poisson process
   vi) Memorylessness property

b) Derive the distribution of interarrival time under a Bernoulli Process and state its expectation and variance (5mks)

c) Let N_1 and N_2 be two independent Poisson processes with rates λ_1 and λ_2 respectively. Let N_t be the merged process $N_t = N_1 + N_2$, find the probability that $N_1 = 2$ and $N_1 = 5$ (5mks)

d) State four examples of stochastic processes in real life (4mks)

e) You get email according to a Poisson process at a rate of $\lambda = 5$ messages per hour. You check your email every thirty minutes.

   i) What is the probability of No new messages (2mks)
   ii) What is the probability of 2 or more messages (2mks)

Question Two (20mks)

a) (i) In a large population of adults, 30% have received CPR training. If adults from this population are randomly selected, what is the probability that the 6^{th} person sampled is the first that has received CPR training. (5mks)

   (ii) What is the probability that the person trained in CPR occurs on or before the 3^{rd} person sampled? (5mks)

b) A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 3/5. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months.

   i) Compute the Probability Mass Function (5mks)
   ii) Calculate the probability that there will be at least four consecutive months in which no accidents occur. (5mks)

Question Three (20mks)

a) For a Poisson process with rate ¸λ = 30 costumers per hour, find

i) The expected number of arrivals in the first 10 minutes of an hour (4mks)

ii) The probability that exactly 4 arrivals in the first 10 minutes of an hour (4mks)

iii) The probability of four or fewer arrivals in the first 10 minutes of an hour (6mks)

b) Consider a system that can be in one of two possible states, $S = \{0 \quad 1\}$. In particular, suppose that the transition matrix is given by

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

Suppose that the system is in state 0 at time n = 0, i. e $X_0 = 0$

i) Draw the state transition diagram (2mks)

ii) Find the probability that the system is in state 0 at time n = 2 (4mks)

Question Four (20mks)

a) Alex takes a multiple choice quiz in his Anthropology 100 class. The quiz has 10 questions, each has 4 possible answers, only one of which is correct. Alex did not study for the quiz, so he guesses independently on every question.
What is the probability that Alex answers exactly 2 questions correctly? (6mks)

b) A radio station gives a pair of concert tickets to the sixth caller who knows the birthday of the performer. For each person who calls, the probability is 0.75 of knowing the performer's birthday. All calls are independent.

(i) What is the PMF of L, the number of calls necessary to find the winner?(3mks)

(ii) What is the probability of finding the winner on the tenth call? (3mks)

(iii) What is the probability that the station will need nine or more calls to find a winner? (3mk)

i) Consider the Markov Chain with three states, $S = \{1, \; 2, \; 3\}$ that has the following transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

If we know $P(X_1 = 1) = P(X_1 = 2) = \frac{1}{4}, find \; P(X_1 = 1, X_2 = 2, X_3 = 1)$ (5mks)

Question Five (20mks)

a) The leading brewery on the West Coast (A) has hired a TM specialist to analyze its market position. It is particularly concerned about its major competitor (B). The analyst believes that brand switching can be modeled as a Markov chain using 3 states, with states A and B representing customers drinking beer produced from the aforementioned breweries and state C representing all other brands. Data are taken monthly, and the analyst has constructed the following one-step transition probability matrix.

A	B	C
0.7	0.2	0.1
0.7	0.75	0.05
0.1	0.1	0.8

What are the steady-state market shares for the two major breweries? (10mks)

b) You are given the following information:

• Mortality for an individual can be described using a non-homogenous Markov Chain processwith two states:

State 1: Alive

State 2: Deceased

You are given the following transition probability matrices for this individual:

$$Q_0 = \begin{pmatrix} 0.9 & 0.1 \\ 0 & 1 \end{pmatrix} \quad Q_1 = \begin{pmatrix} 0.8 & 0.2 \\ 0 & 1 \end{pmatrix} \quad Q_2 = \begin{pmatrix} 0.7 & 0.3 \\ 0 & 1 \end{pmatrix} \quad Q_3 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

- An insurance policy is issued to this individual at time 0.
- The insured is in state 1 at the time the policy is issued.
- A benefit of $100,000 is paid out upon transition of the insured from state 1 to state 2.
- Transitions occur at the end of each time period.
- The insurance company receives a premium of $25,000 at the beginning of each time period,if the insured is in state 1 at that time.
- i = 5%

Calculate the benefit reserve for this policy at time 1, assuming the insured is in state 1 and the premium for this time period has not yet been paid. (10mks)