JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

2nd YEAR 1ST SEMESTER 2022/2023 ACADEMIC YEAR

MAIN

COURSE CODE: WMB 9205

COURSE TITLE: VECTOR ANALYSIS

EXAM VENUE: **STREAM: (BSc./ BEd)**

DATE: 20/12/2022 **EXAM SESSION: 15.00-17.00PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 (30marks) COMPULSORY

a) Evaluate each of the following:
   (i) $2j \times (3j - 4k)$ (4mks)
   (ii) $(i + 2j) \times k$ (3mks)

b) Find the area of the triangle with vertices at (3,-1,2), (1,-1,-3) and (4,-3,1)(6mks)

c) $If\ A = x^2yz\ i - 2xz^3\ j + xz^2k$ and $B = 2z\ i + yj - x^2k, find\ \frac{\partial^2}{\partial x\ \partial y}(A \times B)at\ (1,0,-2)$(6mks)

d) If $\nabla\emptyset = 2xyz^3\ i + x^2z^3\ j + 3x^2yz^2\ k$, find $\emptyset(x, y, z)\ if\ \emptyset(1, -2,2) = 4$. (5mks)

e) If A= $3xyz^2\ i + 2xy^3\ j - x^2yz\ k\ and\ \emptyset = 3x^2 - yz, find\ \nabla \cdot A$(6mks)

f) If F= $5xy - 6x^2)\ i + (2y - 4x)j, evaluate\ \int_C F \cdot dr\ along\ the\ curve\ C\ in\ the\ xy\ plane, y = x^3\ from\ the\ point\ (1,1) to\ (2,8)$. (7mks)

QUESTION 2 (20marks)

(a) Given the vectors $\underline{u} = 3i - 2j + k,\ \ \underline{v} = 2i - 4j - 3k,\ \ \underline{w} = -i + 2j + 2k$, find

(i) $|7\underline{u}|$, (ii) $|-2\underline{u} + 8\underline{v} - \underline{w}|$ (7mks)

(b) If $\emptyset = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5, find\ \nabla^2\emptyset$ (5mks)

(c) Show that the vector $\underset{\sim}{v} = \frac{5}{(x^2 + y^2)}(-y\underset{\sim}{i} + x\underset{\sim}{j})$ is i) incompressible ii) irrotational (6mks)

QUESTION 3 (20marks)

a)Given the vectors $\underline{\alpha} = 5t\,i + t\,j - t\,k, \underline{\beta} = \sin t\,i - \cos t\,j + 2k$, prove that $(\underline{\alpha} \times \underline{\beta}) = -(\underline{\beta} \times \underline{\alpha})$.
[5 marks]

b)For the vectors $\underline{\alpha} = 5t^2\,i + t\,j - t^3\,k, \underline{\beta} = e^{2t}\,i - \cos t\,j + 8k$, compute $\frac{d(\underline{\alpha} \times \underline{\beta})}{dt}$ [5 marks]

c)Find all the turning points for functi0n $\Phi(x, y, z) = 8x^2 + 24y^2 + 16z^2 + 24x + 16z + 1$ (10Amks)

QUESTION 4(20marks)

(a) If a vector field $\underset{\sim}{F}(x, y, z)$ is conservative then show that the work integral $\int_C \underset{\sim}{F} \bullet d\underset{\sim}{r}$ between points A and B is independent of the path c chosen between the two points. (5mks)

(b) Given $\phi(x, y, z) = xy^2z, \underline{a} = xz\,i - xy^2\,j + yz^2\,k$, show that $\frac{\partial^4}{\partial y \partial x^2 \partial z}[\phi\underline{a}]\Big|_{(0,1,1)} = 8i - 8j$ (8mks)

(c) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t, z = 2\sin 3t$ where t is the time. Find its velocity and acceleration at time $t = 0$ (7mks)

QUESTION 5(20marks)

a)State and prove the Green's Theorem. (7 mks)

b)A curve $\frac{x^2}{4}+\frac{y^2}{9} = 1$, is parametrically defined by the vector equation,

$\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j}$

UsingGreen's theorem show that the area enclosed by this curve is 6π (13 mks)