JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE/BACHELOR OF SCIENCE(ACTUARIAL SCIENCE WITH IT)

4^{TH} YEAR 1^{ST}SEMESTER 2022/2023 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WMB 9401
COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS I

EXAM VENUE: AH **STREAM:**

DATE: 6/12/2022 **EXAM SESSION: 9.00-11.00AM**

TIME: 2.00 HOURS

INSTRUCTIONS:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY)

a) State the order and degree of the partial differential equations below

i) $$\frac{\partial^4 y}{\partial x^4} + \left(\frac{\partial y}{\partial x}\right)^5 + \left(\frac{\partial^4 z}{\partial x^4}\right)^2 = 0$$

ii) $$\left(\frac{\partial y}{\partial x}\right)^4 + \frac{\partial^2 z}{\partial x^2} + \left(\frac{\partial z}{\partial x}\right)^3 = 0$$ (4 marks)

b) Give a general form of each of the following equations hence state the difference between them

i) Semi-Linear partial differential Equation

ii) Quasi-linear partial differential equation (6 marks)

c) Solve the simultaneous Differential equation

$$\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2}$$ (6 marks)

d) Determine $f(x, y)$ given the following differential equations

i) $df(x, y) = \frac{xdy + ydx}{xy}$

ii) $df(x, y) = zdx + xdz + y^2 dy$ (4 marks)

e) Find the orthogonal trajectory on the concoid $(x + y)z = 1$ of its intersection with the family of planes $x - y + z = k$ where k is a parameter

(8 marks)

f) State the necessary condition for the Differential equations $f(x, y, z, p, q)$ and $g(x, y, z, p, q)$ s to be compatible (2 marks)

QUESTION TWO (20 marks)

a) Show that the equation $p^2 + q^2 = 1$ and $(p^2 + q^2)x = pz$ are compatible hence find their solution. (8 marks)

b) Solve the Pfaffian differential equation

$(z^2 + 2xy)dx + (x^2 + 2yz)dy + (y^2 + 2xz)dz = 0$ (6 marks)

c) Form a partial differential equation by eliminating the arbitrary function f from the function $x + y + z = f(x^2 + y^2 + z^2)$ (6 marks)

QUESTION THREE (20 marks)

a) By use of an appropriate auxiliary equation solve the equation

$(y^2 + yz + z^2)dx + (z^2 + xz + x^2)dy + (x^2 + xy + y^2)dz = 0$ (6 marks)

b) Find $f(y)$ such that the Pfaffian differential equation

$(y^2 + z^2 - x^2)dx + 2xf(y)dy - 2xzdz = 0$ is integrable hence solve it.

(8 marks)

c) Use Lagrange's method to solve $z(z^2 + xy)(px - qy) = x^4$

(6 marks)

QUESTION FOUR (20 marks)

a) By eliminating the arbitrary constants a and b from $2z = (ax - y)^2 + b$ form a partial differential equation (4 marks)

b) Solve the homogeneous equation using the substituition $x = uz$ and $y = vz$
$2(y+z)dx - (x+y)dy + (2y - x + z)dz = 0$ (10 marks)

c) By choosing appropriate multipliers where necessary solve
$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$$ (6 marks)

QUESTION FIVE (20 marks)

a) Solve the Cauchy's problem for $zp + q = 1$ where the initial data curve is $x_0 = \mu, y_0 = \mu, z_0 = \frac{\mu}{2}$ for $0 \le \mu \le 1$ (8 marks)

b) Use Charpit's method to find the complete integral of $px + qy = pq$ (6 marks)

c) By taking one variable to be a constant solve the total differential equation
$(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$ (6 marks)