QUESTION 1 [30 MARKS]

a. A mortality table, which obeys Gompertz' Law for older ages, has $\mu_{70} = 0.025330$ and $\mu_{90} = 0.126255$. Find the probability that a life aged 60 will survive for 20 years. [5 marks]

b. You are given the survival function $S_0(x) = \frac{1}{(1+x)^2}$ for $x \geq 0$ .Describe the event and calculate or formulate each of the following:

i. ${}_{10}q_{10}$ [2 marks]
ii. ${}_{10|10}q_{10}$ [2 marks]
iii. ${}_{t}p_x$ [2 marks]
iv. ${}_{t|u}q_x$ [2 marks]
v. $P[K_{10} = 10]$ [2 marks]

c. Below is an extract from English life table 15 (males). [2 marks]

Age, x	l_x
58	88,792
59	87,805

Estimate $l_{58.25}$ assuming a uniform distribution of death between exact ages 58 and 59.

d. $\mu_x = 0.0908 + 0.001(x-70)$ for $x \geq 55$ calculate ${}_5q_{60}$ (3 Marks)

e. Let Tx denote the complete future lifetime of a life now aged exactly x. Define in terms of probabilities involving Tx :
i. The survival function, $S_x(t)$ (2 Marks)
ii. The force of mortality, $\mu_{x+t.}$ (2 Marks)
iii. Distribution function, $F_x(t)$ (2 Marks)

f. For a force of mortality μ_x that is known to follow Gompertz' Law, calculate the parameters B and C if $\mu_{50} = 0.017609$ and $\mu_{55} = 0.028359$. (4 Marks)

QUESTION 2[20 MARKS]

a. A clinical trial is being carried out to test the effectiveness of a new drug. Sixty patients were involved in the trial, which followed them for 2 years from the start of their treatment. The following data show the period in complete months from the start of treatment to the end of observation for those patients who died or withdrew from the trial before the end of the 2-year period.

Deaths: 8, 10, 10, 16, 20

Withdrawals: 2, 6, 9, 16, 18, 22, 22

Calculate the Kaplan-Meier estimate of the survival function. (8 marks)

b. Mortality of a group of lives is assumed to follow Gompertz' law. Calculate $\mu_x$for a 30-year oldand a 70-year old, given that $\mu_x$is 0.003 for a 50-year old and 0.01 for a 60-year old. (6 marks)

c. Calculate the exact values of the complete and curtate expectation of life for a newborn animal subject to a constant force of mortality of 0.05 per annum. (6 marks)

QUESTION 3[20 MARKS]

a. A chef specializing in the manufacture of fluffy meringues uses a *Whiskmatic* disposable electric kitchen implement. The *Whiskmatic* is rather unreliable and often breaks down, so the chef is in the habit of replacing the implement in use at a given time, shortly before an important social function or after making the 1,000th fluffy meringue with that implement. The following times until mechanical failure (no asterisk) or replacement whilst in working order (asterisk) were observed (measured in days of use):

17, 13, 15*, 7*, 21, 18*, 5, 18, 6*, 22, 19*, 15, 4, 11, 14*, 18, 10, 10, 8*, 17

i. State the values *n*,*m*,*k*,*tj*,*dj*,*cj*and *nj*for these data, assuming that censoring occurs just after the failures were observed. (4 marks)

ii. Calculate the Kaplan-Meier estimate of the *Whiskmatic* survival function. (8 marks)

iii. Using Greenwood's formula, estimate var[S*(16)] (2 marks)

iv. Calculate the Nelson-Aalen estimate of the cumulative hazard function using the given data values. (6 marks)

QUESTION 4[20 MARKS]

a. You have been asked to investigate whether the rate of ill-health retirement of the employees of a large company varies with their duration of employment. The company's records show:

The date on which an employee was hired
The calendar year in which they retired, if an employee left employment as a result ofill-health retirement
The date of retirement, if an employee reached the normal retirement age of 65
The date of leaving, if an employee left the company for any other reason.
In the context of this investigation consider the following types of censoring and in each case:
Describe the nature of the censoring
State whether or not that type of censoring is present in these data
If that particular type of censoring is present, explain how it arises.

i. Left censoring (3marks)
ii. Right censoring (3marks)
iii. Interval censoring (3marks)
iv. Informative censoring. (3marks)

b. A study has been undertaken into the effect of a new treatment on the survival times of patients suffering from a tropical disease. The following model has been fitted:

$h_i(t)=h_0(t)\exp(\beta^T Z)$

Where $h_i(t)$ is the hazard at time t, where t is the time since treatment

$h_0(t)$ is the baseline hazard at time t

Z is a vector of covariates, where

Z_1 =Period from diagnosis to treatment in years

Z_2 =Period from diagnosis to treatment in years

Z1 =0 if female , 1 if male

B is vector of parameters where β_1= 0.5, β_2= 0.01, β_3= -0.05

i. State the group of lives to which the baseline hazard applies. (1marks)

ii. For a male who was given the new treatment 6 months after diagnosis:

a) Write down the hazard function, in terms of $h0(t)$ only. (2marks)

b) Express the survival function, in terms of $h0(t)$ only. (2marks)

iii. For a female given the new treatment at the time of diagnosis, the probability of survivalfor 5 years is 0.75. Calculate the probability that the male in (ii) will survive 5 years. (3marks)

QUESTION 5[20 MARKS]

a. Explain the differences between random censoring and Type I censoring in the context of an investigation into the mortality of life insurance policyholders. Include in your explanation a statement of the circumstances in which the censoring will be random, and the circumstances in which it will be Type I, and give an example of each. (4marks)

b. Explain what is meant by non-informative censoring in the investigation in (a). Describe a situation in which censoring might be informative in this investigation (4marks)

c. Suppose that a group of 15 laboratory rats are injected with a new drug. They are observed over the next 30 days.The following events occur:

Day	Event
3	Rat 4 dies from effects of drug.
4	Rat 13 dies from effects of drug.
6	Rat 7 gnaws through bars of cage and escapes.
11	Rats 6 and 9 die from effects of drug.
17	Rat 1 killed by other rats.
21	Rat 10 dies from effects of drug.
24	Rat 8 freed during raid by animal liberation activists.
25	Rat 12 accidentally freed by journalist reporting earlier raid.
26	Rat 5 dies from effects of drug.
30	Investigation closes.

Calculate the Kaplan-Meier estimate of the survival function and Kaplan-Meier estimate of the distribution function. (12marks)