**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE**

**SPECIAL RESITS DECEMBER 2022**

**MAIN REGULAR**

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**COURSE CODE: WMB 9205/SMA205**

**COURSE TITLE: LINEAR ALGEBRA II**

**EXAM VENUE:** **STREAM: (Bed/BSc. Actuarial)**

**DATE:** **EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE COMPULSORY (30 MARKS)**

ai)Define the term linearly dependent in relation to linear spaces. (2mks)

ii) Determine if the set $L=\left\{\begin{pmatrix}1 & 1\\ 0 & 3\end{pmatrix},\begin{pmatrix}-1 & 0\\ 2 & 1\end{pmatrix},\begin{pmatrix}-1 & 2\\ 6 & 0\end{pmatrix}\right\}$ is linearly independent in $M_{2X2}$ using standard coordinate mapping.
(6mks)

b i)Write down the characteristic polynomial of matrix $N=\begin{pmatrix}1 & 2\\ 8 & 1\end{pmatrix}$ of linear operator T.

(3mks)

ii)Let $A=\begin{pmatrix}10 & 1\\ 0 & 3\end{pmatrix}, B=\begin{pmatrix}-1 & 0\\ 20 & 1\end{pmatrix}$ .Show that $(AB)^t = B^t A^t$ (3mk)

c) Let $V = \mathbb{R}^2$ and $W = \{(x, y) \in \mathbb{R}^2 : 2x - y = 0\}$. Show that $W$ is a subspace of $V$

(3mks)

d) What is a symmetric matrix $A=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}$ (2mk)

e) Diagonalize the matrix$A = \begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$ (4mks)

f) Define the term linear mapping (3mks)

g)Given that $\beta=\left\{\begin{pmatrix}3\\1\end{pmatrix},\begin{pmatrix}3\\-1\end{pmatrix}\right\}$, $\alpha=\left\{\begin{pmatrix}1\\4\end{pmatrix},\begin{pmatrix}4\\1\end{pmatrix}\right\}$, $and\ N=\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$ are ordered bases for $R^2$ show that the transition matrix from $\beta$ to $\alpha$ i.e. $\alpha P_\beta = (\alpha P_N)({}_N P_\beta)$ (4mks)

## QUESTION TWO (20 MARKS)

a) Let $A = \begin{pmatrix} 10-i & 1 \\ 4+i & -3i \end{pmatrix}$, find the

i)conjugate of matrix $A$ (3mks)

ii) Conjugatetranspose of matrix $A$ (3mks)

iii) Hence evaluate $A + A^t\left[A + \overline{A}^t\right]^t$ (3mks)

b) Let $\beta = \{X, Y, Z\}$ *be a basis for* $R^3$ .If $X = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, Y = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, Z = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ , and given , $W = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

find $[W]_\beta$ . (3mks)

c) Let $A$ be a square matrix. Show that $A$ is orthogonal if and only if $A^T$ is orthogonal (3mks)

d) Find the matrix of linear transformation defined by

$T(x_1, x_2, x_3) = (x_1 + x_3,\ x_1 - x_2,\ x_3 + x_2)$with respect to the standardbasis (3mks)

e).Let $T : P_2 \rightarrow P_2$ be a linear transformation defined by

$$T(a + bx + cx^2) = -2c + (a + 2b + c)x + (a + 3c)x^2$$

Find the matrix associated with $T$ with respect to the standard basis $B = \{1, x, x^2\}$ (2mks)

## QUESTION THREE (20 MARKS)

a) Define

(i) an eigenvalue (1mks)

(ii) an eigenvector of matrix $A$ in $\mathbb{R}^n$ (3mks)

b) Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

i) Obtain the characteristic equation of $A$ (4mks)

ii) Find the eigenvalues of $A$ (4mks)

iii) Show that $A$ satisfies the Cayley-Hamilton theorem (4mks)

iv) Find the inverse of $A$ using Cayley-Hamilton theorem (4mks)

## QUESTION FOUR (20 MARKS)

a) i) Define term isomorphism (1mk)

ii) If $T$ is an isomorphism, show that $T^{-1}$ is also an isomorphism (2mks)

b) Let $M = \begin{pmatrix} -1 & 0 \\ 10 & 1 \end{pmatrix}$ be the matrix of linear operator $T$ defined on $R^2$.

i) Define rule $T$ on a general vector $\begin{pmatrix} x \\ y \end{pmatrix}$ in $R^2$ (6mks)

i)i Show that $T$ is invertible in $R^2$ (3mks)

iii) Define rule $T^{-1}$ the inverse of $T$ ,on a general vector $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ in $R^2$ (5mks)

iv)Verify that $T^{-1}$ is the true inverse. (3mks)

## QUESTION FIVE (20 MARKS)

Let $B = \begin{pmatrix} -8 & 2 & 3 & -1 \\ -7 & 1 & 3 & -1 \\ -6 & 2 & 1 & -1 \\ -5 & 2 & 3 & -4 \end{pmatrix}$ be a matrix of linear operator $T$ defined $R^4$.

i) Discover if $u = [1,1,1,1]^t$, $v = [1,1,1,0]^t$, $w = [2,5,2,2]^t$; are eigenvectors of $B$. (10mks)

ii) Suppose $\lambda_u, \lambda_v, \lambda_w, \lambda_\circ$ are the associated eigen values of matrix $B$ then

Show that $\lambda_u + \lambda_v + \lambda_w + \lambda_\circ = traceB$ (5mks)

iii)Discuss exhaustively diagonal properties of $B$ (5mks)