JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCES

SPECIAL RESITS DECEMBER 2022

MAIN CAMPUS

COURSE CODE: WMB9414

COURSE TITLE: FOURIER ANALYSIS

EXAM VENUE: **STREAM: BED AND ACTUARIAL**

TIME: 2 HOURS   **EXAM SESSION:**

<u>**Instructions:**</u>

1. Answer question one (compulsory) and any other two questions.
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION 1 (30 MARKS)

a) Find the limit $\lim\limits_{x\to 0}\left\{\dfrac{\cos(x^4)-1+\dfrac{1}{2}x^8}{x^6}\right\}$ (5 marks)

b) Determine whether the given functions are even, odd or neither

i) $f(x)=\sin\left(\dfrac{n\pi x}{L}\right)$ on $-L\le x\le L$

ii) $f(x)=\cos\left(\dfrac{n\pi x}{L}\right)$ on $-L\le x\le L$

iii) $f(x)=x^2-1+e^{2x}$ on $-L\le x\le L$ (9 marks)

c) Compute the Maclaurin series as far as x^6 term for the following functions

i) $\dfrac{\sin(x)}{x}$

ii) $\dfrac{\sin(x^2)}{x^2}$ (8 marks)

d) The Fourier series of the function f defined by $f(x)=x^2$ on the interval $[-\pi,\,\pi]$ is known to be convergent.

What do you understand by

i)   Period of f
ii) f is Periodic
iii)   Periodic extension of f
iv)   Fourier Coefficients of expansion

Give a Sketch Graph of two periodic extensions of f (8 marks)

QUESTION 2 (20 MARKS)

Solve the heat equation $u_t = \alpha^2 u_{xx}$, $0 < x < 1$, $t > 0$ with the Dirichlet Boundary conditions $u(t,0) = u(t,1) = 0$, $t > 0$ and initial conditions $u(0,x) = g(x) = x$, $0 \leq x \leq 1$

(20 marks)

QUESTION 3 (20 MARKS)

Find the Fourier series of the function defined in pieces (piecewise constant function) by

$$f(x) = \begin{cases} 8 & 0 < x < 2 \\ -8 & 0 < x < 4 \end{cases}$$

where f is periodic with period 4. What does the series converge to at

i) $x = 2$
ii) $x = -3$ (20 marks)

QUESTION.4 [20 marks]

(a) Given the voltage $v = f(t)$ $volts$, and $i = F(t)$ $amperes$, such that
$v = 12.0 + 5.2\cos wt + 2.4\cos 2wt + 0.9\cos 3wt + ... + 2.7\sin wt + 1.8\sin 2wt + 0.2\sin 3wt + ...$
$i = 8.50 + 4.1\cos wt + 2.0\cos 2wt + 0.6\cos 3wt + ... + 3.6\sin wt + 1.2\sin 2wt + 0.3\sin 3wt + ...$
find the average value of power vi in $watts$, over one cycle.   [9 marks]
(b) For the function $f(x) = \{2x \qquad -5 < x < 5$
(i) sketch graph of $f(x)$ over the interval $-20 < x < 20$
(ii) state period of $f(x)$
(iii) obtain Fourier series for $f(x)$ [11 marks]

QUESTION 5 [20 marks]

One cycle of a periodic waveform $y = f(x)$ of period 2π is defined by the below data.

x^0	0	30	60	90	120	150	180	210	240	270	300	330
$y(x)$	15	20	23	24	20	8	3	4	9	12	10	11

Determine the approximate Fourier series for $y = f(x)$ up to and including the third harmonic.

[20 marks]