JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

SPECIAL RESITS DECEMBER 2022

MAIN CAMPUS

COURSE CODE: WMB9105/SMA103

COURSE TITLE: LINEAR ALGEBRA 1

EXAM VENUE: **STREAM: BED AND ACT SCIENCE**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Define the following terms as used in Linear Algebra
   i) Linearly Independent Vectors (1mark)
   ii) Linearly Dependent Vectors (1mark)
   iii) Orthogonal Vectors(1mark)
   iv) Spanning Set (1mark)

b) Let A be the following matrix $\begin{pmatrix} 1 & 3 \\ -2 & -8 \end{pmatrix}$. Compute the matrix
   i) A^2(2marks)
   ii) AA^T(3marks)
   iii) A^{-1} (1marks)
   iv) Find numbers p and q such that $A^2 = pA + qI$, where I isthe2×2 identity matrix. (4marks)

c) Consider the set $\{u_1 = (-2,4,1), u_2 = (1,2,3)\}$ of vectors in $\mathbb{R}^3$. Determine its dimension. (5marks)

d) Given $\tilde{u} = (2, -5, -1)$ and $\tilde{v} = (-7, -4, 6)$. Find
   i) $3\tilde{u} + \tilde{v}$ (2marks)
   ii) the distance between $\tilde{u}$ and $\tilde{v}$(2marks)
   iii) normalize each vector (4marks)

e) Prove that if a, b and c are vectors such that $a + b = a + c$ then,$b = c$ (3marks)

QUESTION TWO (20 MARKS)

a) Let $\tilde{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\tilde{v}_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ and $\tilde{v}_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$.
   i) Determine if the set $\{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3\}$ is linearly independent (4marks)
   ii) If possible, find a linearly dependence relation among $\tilde{v}_1, \tilde{v}_2$ and $\tilde{v}_3$. (6marks)

b) Express the polynomial $p = 6 + 11x + 6x^2$ as a linear combination of $p_1 = 2 + x + 4x^2, p_2 = 1 - x + 3x^2$ and $p_3 = 3 + 2x + 5x^2$ (10marks)

QUESTION THREE (20 MARKS)

a) i) What is a Vector Space (2mks)
ii) Determine whether the function $x - 3y = 1$ constitute a vector space (4marks)

b) Let W be a vector space, $\tilde{w}$ a vector in W, $\breve{0}$ the zero vector in W, α a scalar and 0 the zero scalar. Prove that

i) $0\breve{w} = 0.$ (2marks)

ii) $\alpha\breve{0} = \breve{0}$ (2marks)

c) Let $\tilde{a} = \begin{pmatrix} 0 \\ -5 \\ 2 \end{pmatrix}$ and $\tilde{b} = \begin{pmatrix} -4 \\ -1 \\ 8 \end{pmatrix}$. Find

i) $\tilde{a}.\tilde{b}$ (2marks)

ii) $\tilde{a} \times \tilde{b}$ (3marks)

iii) The angle between $\tilde{a}$ and $\tilde{b}$ (3marks)

iv) Projection $\tilde{a}$ onto $\tilde{b}$ (2marks)

QUESTION FOUR (20 MARKS)

a) i) Let $\tilde{v} \in W$ such that $\tilde{v} = (c,\ c^2,\ d)$. Consider the subset W of $\mathbb{R}^3$ consisting of the vectors of the form $(c,\ c^2,\ d)$, where the second component is the square of the first. Determine whether W is a subspace of $\mathbb{R}$. (5marks)

ii.)Prove that if U and W are subspaces of V, then the union$U \cup W$ is a not generally a subspace of V. (5marks)

b) Prove that for any set $S$of vectors in$W$, the set span $(S)$is a subspace of W(10marks)

QUESTION FIVE (20 MARKS)

a) Determine whether$V = \{v_1 = (1,-1,1), v_2 = (0,1,2), v_3 = (3,0,-1)\}$is a basis for $\mathbb{R}^3$. (5marks)

b) Consider the plane through the point (1,1,2) and perpendicular to the vector product of $u = (1,2,3)$ and $v = (3,0,1)$. Find the equation of the plane.(5marks)

c) i) What is a linear transformation? (2marks)
ii) Determine whether the function $S: \mathbb{C}^2 \rightarrow \mathbb{C}^2$ that swaps vector components $S\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$ is a linear transformation (8marks)