JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

SPECIAL RESITS DECEMBER 2022

MAIN CAMPUS

COURSE CODE: WMB9302
COURSE TITLE: COMPLEX ANALYSIS

EXAM VENUE: **STREAM: BED SCIENCE**

DATE: 1/12/2022 **EXAM SESSION: 2.00-4.00PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) – 30 MARKS

a) Define each of the following terms as used in Complex Analysis
   i) Principal argument
   ii) Complex limit (4 marks)

b) Sketch the disk represented by $0 < |Z - 2| \leq 2$ hence
   i) State the deleted neighbourhood (4 marks)
   ii) State any two boundary points (1 mark)
   iii) State any two points in the neighbourhood of the disk (1 mark)

c) Compute the nthroot for the $\left(\sqrt{3} - i\right)^{\frac{1}{3}}$, hence sketch an appropriate circle indicating the roots w_0, w_1, and w_2. (4 marks)

d) Find the image of a line $x = 3$ under the complex mapping $w = z^2$ for $w, z \in \mathbb{C}$, hence sketch the line and its image under the mapping (4 marks)

e) Express $1 - i$ in exponential form using the principal argument. (4 marks)

f) Describe all the transformations represented by a complex mapping $f(z) = \sqrt{2}iz - 2 + 3i$ (4 marks)

g) Evaluate the line integral $I = \oint_c (xdx + ydy)$ where C comprises the triangle $O(0,1)$, $A(1,2)$ and $C(0,0)$ (4 marks)

QUESTION TWO (20 MARKS)

a) Prove that if a Complex Function $f(z) = u(x, y) + iv(x, y)$ is analytic at any point z, and in the domain D, then the Laplace's Equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, can be verified. (6 marks)

b) Find the derivative of $\frac{iz}{3z^2 i + 1}$ (4 marks)

c) Solve for w, given the complex function $e^w = \sqrt{3} + i$ for $w, \in \mathbb{C}$. (6 marks)

d) Compute the principal value of the complex logarithm $\ln z$ for $z = 2 + i$ (4 marks)

QUESTION THREE (20 MARKS)

a) State and prove De-Moivre's Theorem hence use it to evaluate $\left(\sqrt{2} + i\right)^5$, giving your answer in the form $a + bi$, $a, b \in \mathbb{R}$ (8 marks)

b) Show that the nth of unity are given by $(1)^{\frac{1}{n}} = \cos\frac{2k\pi}{n} - i\sin\frac{2k\pi}{n}, k = 0,1,2,.....(n-1)$ hence exaluate the cube root of unity (6 marks)

c) Use the definition of the derivative of a complex function to determine the derivative of $f(z) = z^2 - 2z$ in the region where the derivative exists.

(6 marks)

QUESTION FOUR (20 MARKS)

a) Find the value of $(1 + \sqrt{3}i)^i$ (5 marks)

b) Given that $e^{i\theta} = \cos\theta + i\sin\theta$ for any Real Number θ, prove that $e^{iz} = \cos z + i\sin z$ for any complex number *z*. (5 marks)

c) Solve the compex quadratic equation $iz^2 - z + i = 0$

(5 marks)

d) Evaluate the integral $\oint_c \frac{z}{z^2 + 25} dz$, where *C* is the circle $|z - 2i| = 4$ using the Cauchy's integral formular. (5 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate $\oint \frac{1}{z} dz$, where *C*is the circle $x = \cos t$, $x = \sin t$ for $0 \le t \le 2\pi$

(4 marks)

b) Given $z_1 = \left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$ and $z_2 = \sqrt{3}\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$ determine the value of

a) $z_1 z_2$ b) z_1/z_2

giving your answer in the form $a + bi$ (4 marks)

c) Stae L'Hopital's Rule and use it to compute

$$\lim_{z \to 1+i} \frac{z^5 + 4z}{z^2 - 2z + 2}$$ (6 marks)

d) Given the complex function $f(z) = u(x, y) + iv(x, y)$, verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic hence find $v(x, y)$ the harmonic conjugate *u*,Hence find the corresponding analytic function $f(z) = u + iv$. (6 marks)