JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN

ACTUARIAL SCIENCE WITH IT

3rd YEAR 1st SEMESTER 2022/2023 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: WAB 2315

COURSE TITLE: TIME SERIES ANALYSIS

EXAM VENUE: **STREAM: (BSc. Actuarial Science)**

DATE: **EXAM SESSION: Sept-Dec 2022**

TIME: 2.00 HOURS

Instructions:

i. Answer questions one and any other two.
ii. Candidates are advised not to write on the question paper.
iii. Candidates must hand in their answer booklets to the invigilator while in the examination room.
iv. Candidates are advised to carry their personal computers with R installed beforehand

QUESTION ONE (30 marks)

a) Define the following
   i. Stationary time series process (1 mark)
   ii. Invertible time series process (1 mark)
   iii. Causal time series process (1 mark)

b) Suppose that the time series $\{x_t, t \in z\}$ is given by

$$X_t = \varepsilon_t + \varphi\, \varepsilon_{t-1} \qquad \text{where } \varepsilon_{t-1} \sim WN(0, \sigma^2)$$

Determine the expected value and autocorrelation function for the process (6 marks)

c) Simulate a MA (1) process using R with parameter estimate given by 0.7 and plot
   i. Y (3 marks)
   ii. ACF (3 marks)
   iii. PACF (3 marks)
   iv. What is the general pattern observation? (1 mark)

d) State and explain the four different components of time series (4 marks)

e) Suppose

t	x_t
1	1
2	3
3	7
4	9

   i. Find the trend (m_t) and seasonal component (s_t) given $X_t = m_t + s_t + \varepsilon_t$ (3 marks)
   ii. Use an appropriate graphical technique to represent question (i) (2 marks)

f) Define the following
   i. A strictly stationary process (1 mark)
   ii. A weakly stationary process (1 mark)

QUESTION TWO (20mks)

Consider the following Time Series data 3,5,7,10,15.

i. Plot autocorrelation function (ACF)
ii. Plot partial autocorrelation function (PACF) for the data
iii. Using (i) and (ii) classify the time series process

QUESTION THREE (20mks)
Use the data provided below to answer the following questions

t	x_t
1	1
2	3
3	7
4	9

a. Fit a polynomial regression of order 2 trend to this data (7 marks)
b. How well is the polynomial regression of order 2 fit the data (2 marks)
c. Fit a simple linear regression trend (7 marks)
d. How well is the linear regression fit the data (2 marks)
e. Compare result in (b) and (d) (2 marks)

QUESTION FOUR (20mks)
The data in the table below are quarterly sales figures at the university eatery during the period 2001-2004.

Year	2001	2002	2003	2004
Quarter	1 2 3 4	1 2 3 4	1 2 3 4	
Sales	1 4 3 2	2 7 5 4	5 7 10 4	

a. Predict the sales figures during the 1st, 2nd and 3rd quarters of the year 2004 using classical decomposition model of the time series $\{x_t\}$ given by $X_t = m_t + s_t + \varepsilon_t$ Where m_t, s_t, and ε_t are the trend, seasonal and noise components respectively. (10 marks)
b. Show that the process $X_t - 0.7X_{t-1} + 0.1X_{t-2} = \varepsilon_t$ is a casual invertible process. Hence or otherwise, determine its autocovariance and partial autocovariance at lag 2. (10 marks)

QUESTION FIVE (20mks)
a) Determine which of the following ARMA process is casual and which of them is invertible (in each case $\{\varepsilon_t\}$ denotes a white noise).
i. $X_t = 0.45X_{t-1} - 0.05X_{t-2} + \varepsilon_t - 0.2\ \varepsilon_{t-1}$ (4 marks)
ii. $X_t = 15X_{t-1} + 50X_{t-2} + \varepsilon_t + 0.2\ \varepsilon_{t-1} - 0.48\ \varepsilon_{t-2}$ (4 marks)

b) The data presented below refers to UK outward passenger movements by sea (in 000,000)

	Year 1				Year2				Year3			
Quarter of the year	1	2	3	4	1	2	3	4	1	2	3	4
No of passengers	2.2	5	7.9	3.2	2.9	5.2	8.2	3.8	3.2	5.8	9.1	4.1

(i) Draw a time series plot displaying the 4 quarterly cycle. (2 marks)
(ii) Explain the trend and cyclic pattern (4 marks)
(iii) Assuming linear trend, fit a regression line (6 marks)