# JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES <br> UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OFSCIENCE IN <br> ACTUARIAL SCIENCE WITH IT <br> $4^{\text {th }}$ YEAR $1^{\text {st }}$ SEMESTER 2022/2023 ACADEMIC YEAR <br> MAIN REGULAR 

## COURSE CODE: WAB2413

COURSE TITLE: SURVIVAL ANALYSIS

## EXAM VENUE: STREAM: (BSc. Actuarial Science)

DATE: 13/12/2022 EXAM SESSION: 15.00-17.00PM

## TIME: 2.00 HOURS

## Instructions:

i. Answer questions one and any other two.
ii. Candidates are advised not to write on the question paper.
iii. Candidates must hand in their answer booklets to the invigilator while in the examination room.

## QUESTION ONE (30 marks)

a) State and explain four types of censoring (4 marks)
b) Define the uniform distribution of deaths
c) Given an exponential distribution

$$
f(t)=\left\{\begin{array}{lr}
\lambda e^{-\lambda t}, & \lambda>0, \quad t>0 \\
0, & \text { elsewhere }
\end{array}\right.
$$

i. Find $\mathrm{s}(\mathrm{t})$
ii. Find $h(t)$
d) Explain any four types of life tables stating the cohort and nature of attrition.
e) Given $t P_{x}=\frac{100-x-t}{100-x}$ for $0 \leq \mathrm{x} \leq 100$ and $0 \leq \mathrm{t} \leq 100-\mathrm{x}$

Calculate $\mu_{45}$
f) You are provided with the following extract from a life table,

| x | $\mathrm{l}_{\mathrm{x}}$ |
| :--- | :--- |
| 50 | 99813 |
| 51 | 97702 |
| 52 | 95046 |

Calculate ${ }_{0.75} \mathrm{P}_{50.5}$
(5 marks)

## QUESTION TWO (20mks)

A group of 15 laboratory rats are injected with a new drug. They are observed over the next 30 days. The following events occurs.

| Day | Event |
| :--- | :--- |
| 3 | Rat 4 dies from the effect of the drug |
| 4 | Rat 13 dies from the effect of the drug |
| 6 | Rat 7 dies from the effect of the drug |
| 11 | Rat 6 and 9 dies from the effect of the drug |
| 17 | Rat 1 is killed by other rats |
| 21 | Rat 10 dies from the effect of the drug |
| 24 | Rat 8 is freed during raid by animal laboratory activist |
| 25 | Rat 12 is accidentally freed by journalists reporting earlier raid |
| 26 | Rat 5 dies from the effect of the drug |
| 30 | Investigation closes and remaining rats holds a street party |

Using the Kaplan Meier method,
a) Estimate $\mathrm{s}(\mathrm{t})$ and $\operatorname{var}(\mathrm{S}(\mathrm{t}))$ for the problem.
b) Represent question graphically.
c) Compare and contrast the results using a Nelson Aalen method

## QUESTION THREE (20mks)

Consider the following data from two groups x and y

| x | 3 | 5 | 7 | $9+$ | 18 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 12 | 19 | 20 | $20+$ | $33+$ |

Test the hypothesis $\mathrm{H}_{0}: \mathrm{S}_{\mathrm{x}}(\mathrm{t})=\mathrm{S}_{\mathrm{y}}(\mathrm{t})$ vs

$$
\mathrm{H}_{0}: \mathrm{S}_{\mathrm{x}}(\mathrm{t}) \neq \mathrm{S}_{\mathrm{y}}(\mathrm{t})
$$

## QUESTION FOUR (20mks)

a) Show that

$$
h(t)=h_{0}(t) e^{\beta^{\prime} x} \text { for a cox proportional hazard model.(8 marks) }
$$

b) The following table refers to the survival times (in years) of the patients in a certain chemical trial. An asterisk indicates that the observation was censored. It is assumed that censoring was done at random and that the patients were randomly assigned to the treatment or control arms.

| Control arm | 3 | $6^{*}$ | 7 | $8^{*}$ | $8^{*}$ | $8^{*}$ | 9 | 11 | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Treatment arm | $4^{*}$ | 5 | $8^{*}$ | $8^{*}$ | $8^{*}$ | $8^{*}$ | 10 | $12^{*} 14^{*}$ |  |

Assuming a proportional hazards model, find the estimate of the $\log$ hazard ratio $\beta$ using the cox proportional hazard methods.
(12 marks)

## QUESTION FIVE (20mks)

a) Given ${ }_{\mathrm{t}} P_{x}=1-\left(\frac{t}{100}\right)^{1.5}$ for $\mathrm{x}=60$ and $0<\mathrm{t}<100$.

Calculate $E(T(x))$
(10 marks)
b) Given $\mu_{x+t}=\frac{1}{85-t}+\frac{3}{105-t}$ for $0 \leq \mathrm{t}<85$

Calculate ${ }_{20} P_{x}$
(10 marks)

