JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2^{ND} YEAR 2^{ND} SEMESTER 2022/2023

MAIN REGULAR

COURSE CODE: SPB 9224

COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS II

EXAM VENUE: **STREAM: (BED SCI)**

DATE: **EXAM SESSION:**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) in Section A and ANY other 2 questions in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

SECTION A (COMPULSORY)

Question 1 (30 marks)

(a) Express in polar form the complex number $1 - i$. (3 marks)

(b) Find the total differential of $z(x, y) = 5x^3 - 4x^{2y} + 3y^3$. (2 marks)

(c) Use Cramer's rule to solve for α and β in the following equations:

$$\alpha + 2\beta = 1$$
$$-\alpha + 3\beta = 4$$

(3 marks)

(d) For each of the vectors $\vec{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, investigate whether the vector is an eigenvector of the matrix $A = \begin{pmatrix} 0 - 2 \\ 2 - 4 \end{pmatrix}$. (4 marks)

(e) Given that $f(x, y) = x^3 y - e^{xy}$, find f_{xy}. (3 marks)

(f) Solve for y in the ordinary differential equation

$$\frac{dy}{dx} = 3x - \frac{1}{x}y$$

given that when $x = 2, y = 3$. (3 marks)

(g) Distinguish between Fourier series and Fourier transform, and give physical situations where each may be applied. (4 marks)

(h) Find the inner product of the vectors $\vec{a} = (3i, 1 - i, 2 + 3i, 1 + 2i), \vec{b} = (-1, 1 + 2i, 3 - i, i)$. (2 marks)

(i) Evaluate the determinant of the matrix

$$B = \begin{pmatrix} 0 - 2 \\ 7 - 3 \\ 4 - 1 \end{pmatrix}.$$

(4 marks)

(j) Distinguish between an isolated singularity and a pole. (2 marks)

SECTION B (Attempt any TWO questions from this section)

Question 2 (20 marks)

(a) Determine the partial second order derivatives of $f(x, y) = x\cos y + ye^x$, hence show that the second partial derivative is a commutative operation. (8 marks)

(b) Show that the function $f(z) = e^{-y}\cos x + ie^{-y}\sin x$ satisfies the Cauchy-Riemann conditions, hence find $f'(z)$. (6 marks)

(c) Find the linearization of $f(x, y) = x^2 - 2xy + \frac{1}{4}y^2 + 3$ at the point (1, 2). (6 marks)

Question 3 (20 marks)

(a) For the matrix

$$A = \begin{pmatrix} 0\ -2 \\ 2 \\ 0 \end{pmatrix},$$

(i) find the eigenvalues of A. (6 marks)

(ii) hence find a matrix P that diagonalizes A. (10 marks)

(b) The equation of motion for a mass oscillating at the end of a spring is given by

$$m\frac{d^2y}{dt^2} = -ky$$

where m and k are constants. Solve this equation for y (you may take $k/m = \omega^2$). (4 marks)

Question 4 (20 marks)

(a) For the set S of basis vectors

$$\vec{A} = [0,2,0,0], \vec{B} = [3,-4,0,0], \vec{C} = [1,2,3,4]$$

use the Gram-Schmidt process to transform S to an orthonormal basis for $\mathbb{R}^4$.

(9 marks)

(b) (i) Describe a physical quantity that can be represented by the non-periodic function

$$f(x) = \begin{cases} 1, -1 < x < 1 \\ 0, |x| > 1 \end{cases}.$$

(2 marks)

(ii) Represent the function in (b)(i) above as a Fourier integral. (9 marks)

Question 5 (20 marks)

(a) Find the determinant of the matrix

$$\begin{pmatrix} 0 \\ 2 \\ 0 \\ 0\ -1 \end{pmatrix}.$$

(6 marks)

(b) Solve the following system of equations using Cramer's rule.

$$x_1 + x_2 + x_3 = 3$$
$$3x_2 - x_3 = -2$$
$$2x_1 - x_3 = 0$$

(10 marks)

(c) For the vectors $\vec{u}$ and $\vec{v}$ defined by $\vec{u} = \{[-1,0],[1,2i],[3,-i][0,i]\}$ and $\vec{v} = \{[4,-2i],[2,-i],[1,0][-2,i]\}$, determine $\langle\vec{u},\vec{v}\rangle$. (4 marks)