JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

3RD YEAR 2ND SEMESTER 2022/2023

MAIN REGULAR

COURSE CODE: SPB 9309

COURSE TITLE: CLASSICAL MECHANICS

EXAM VENUE: **STREAM: (BED SCI)**

DATE: **EXAM SESSION:**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) in Section A and ANY other 2 questions in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

Useful constants

gravitational acceleration, g, = 9.8 m/s^2

velocity of light in free space = 3.0×10^8 m/s
mass of an electron = 9.11×10^{-31} kg

Question 1 (30 marks)

(a) Explain the meaning of the following terms:
(i) Generalized coordinate
(ii) Holonomic constraint (2 marks)

(b) An observer on earth sees a spaceship at an altitude of 4360 km moving towards the Earth with a speed of 0.980*c*, where *c* is the speed of light in free space. Find the distance from the space ship to the Earth as measured by the captain of the spaceship. (3 marks)

(c) Due to a force field, a particle of mass 3 kg moves along a space curve whose position vector is given as a function of time by $\vec{r} = (3t^2 + t)\hat{i} + (2t^4 - t^2 + 6)\hat{j} - 12t^2\hat{k}$. Find:
(i) the velocity
(ii) the momentum
(iii) the acceleration of the particle at any time *t*. (7 marks)

(d) Write down Hamilton's equations for the Hamiltonian

$$H(t, p, q) = \frac{p^2}{2m} - mCtq$$

where *C* is a constant. (2 mks)

(e) Determine the energy required to give an electron a speed of 0.94c, starting from rest. (3 mks)

(f) For an Atwood machine, determine the equation of motion using the Newtonian approach, and state any assumption made. (4 marks)

(g) A particle of mass *m* is subjected to a constant force $\vec{F} = m\vec{g}$.The particle moves from rest at a height $y = h$.Find *y* as a function of time. (4 mks)

(h) State the principle of Galilean transformation and explain its limitations under relativistic conditions. (2 mks)

(i) Write down the Lagrange's equations and explain the meaning of all the terms. (3 mks)

Question 2 (20 marks)

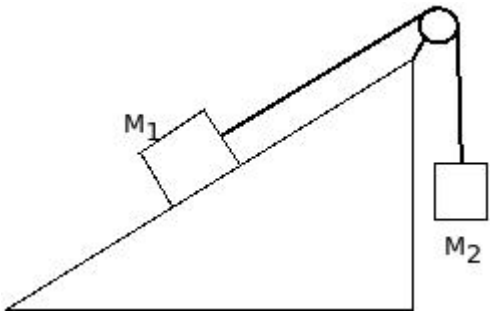
(a) (i) Explain what you understand by a conservative force field and give an example of such a field. (2 mks)

(ii) A 20 kg object is acted on by a conservative force given by $F = -3.0x - 5.0x^2$,with *F* in newtons and x in meters. If the potential energy associated with the force is zero when the object is at

x = 0, what is the potential energy associated with the system when the object is at x = 2.0 m?
(6 marks)

(iii) Investigate whether the force field defined by $\vec{F} = (2y + 8xy^3)\hat{i} + (2x + 12x^2y^2)\hat{j}$ is conservative. (6 mks)

(b) Two masses are connected by a massless string which runs over a massless pulley as shown in the figure below. The coefficient of kinetic friction on the inclined surface is μ, and the angle of inclination to the horizontal is θ. Assuming that M_1 moves down the plane, find the acceleration of the masses.
(6 marks)



Question 3 (20 marks)

(a) (i) Explain the significance of the Hamiltonian. (2 mks)
(ii) Derive the Hamilton's equations. (8 mks)

(b) Use the Hamiltonian method to determine the equation of motion of an Atwood machine.
(10 mks)

Question 4 (20 marks)

(a) Explain, using relevant equations, how the principal of virtual work leads to d'Alembert's principle.
(6 marks)

(b) For the double pendulum,

(i) obtain expressions for the kinetic and potential energies in terms of suitable generalized coordinates. (4 marks)

(ii) hence determine Lagrange's equations for the system. (10 marks)

Question 5 (20 marks)

(a) State the postulates of special relativity . (2 marks)

(b) At age 22, twin A sets off for a distant planet P which is 20 light-years away, leaving behind twin B on earth. Twin A flies in a rocket which can attain a speed 0.95c. Immediately he reaches P, he turns back and flies to earth.

(i) Considering himself to be at rest throughout the voyage, calculate the time determined by A to have elapsed when he arrives back on earth, hence find the supposed age of B. (5 marks)

(ii) Calculate the time that has elapsed as calculated by B, hence the age of A. (3 marks)

(iii) The ages obtained by the two are different. Explain this paradox. (2 marks)

(c) Muons have a mean lifetime of 2.2×10^{-6} s when at rest. They are produced at an altitude of 10 km and travel at $0.995c$ toward the earth. Find:

(i) The mean lifetime measured on earth.

(ii) The time taken to reach ground level in the earth frame.

(iii) The time taken to reach ground level in the particle's frame. (8 mks)