JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2022/2023 EXAMINATIONS

MAIN SPECIAL

COURSE CODE: SPB 9327
COURSE TITLE: QUANTUM MECHANICS I
EXAM VENUE: **STREAM: EDUCATION**
DATE: **EXAM SESSION:**
TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants
$\hbar = 1.054 \times 10^{-34}$Js
mass of proton 1.67×10^{-27}kg
1 eV = 1.6 x 10^{-19} J
mass of electron = 9.11 x 10^{-31} kg

Question 1 (30 marks)

(a) (i) Calculate the de Broglie wavelength for an electron having kinetic energy of 1 eV. (3 marks)

(ii) In the double-slit experiment, two waves defined by $\psi_1 = \frac{1}{\sqrt{2}} e^{ix}$ and $\psi_2 = e^{ix}$ pass through the slits. Determine the probability density on the screen. (4 marks)

(b) Explain the probabilistic interpretation of quantum mechanics. (2 marks)
(c) Derive the time-independent Schroedinger equation. (4 marks)
(d) Define the following terms as used in quantum mechanics:
(i) scattering state. (1 mark)
(ii) tunnelling. (1 mark)

(e) The expectation value of the position of a particle described by the wave function $\psi = \frac{1}{2} x$ limited to the x-axis between x = 0 and x = b is 16. Find the value of b. (3 marks)

(f) A 1 eV electron is trapped inside the surface of a metal. If the potential barrier is 4.0 eV and the width of the barrier is 2 Å, calculate the probability of its transmission. (4 marks)

(g) An eigenfunction of the operator $\frac{d^2}{dx^2}$ is $\psi = e^{2x}$. Find the corresponding eigenvalue.

(3 marks)

(h) An electron has a speed of 500 m/s with an accuracy of 0.004%. Calculate the certainty with which we can locate the position of the electron. (4 marks)

(i) State one postulate of quantum mechanics. (1 mark)

Question 2 (20 marks)

(a) Solve the one-dimensional time-independent Schrödinger equation for a particle in an infinite one-dimensional square well, hence sketch the first three stationary states. (15 marks)

(b) A particle of mass m, confined to a harmonic oscillator potential $V = mx^2\,\omega^2/2$, is in a state described by the wave function

$$\Psi(x,t) = Ae^{\left(\frac{-mx^2\omega}{2\hbar} - i\frac{\omega t}{2}\right)}$$

Verify that this is a solution of the Schrödinger equation. (5 marks)

Question 3 (20 marks)

(a) A Gaussian wave packet is given by $\phi(k) = Aexp\left[- a^2\,(k - k_0)^2/4\right]$ where A is a normalization factor.

(i) Determine A, hence find $\psi(x, 0)$. (8 marks)

(ii) Calculate the probability of finding the particle in the region $-a/2 \leq x \leq a/2$.

(8 marks)

(b) Derive an expression for the dispersive relation. (4 marks)

Question 4 (20 marks)

(a) The Schroedinger equation can be expressed as Obtain an expression for the ground state, hence the energy of the n^{th} state. (7 marks)

(b) Using the uncertainty principle, show that the lowest energy of an oscillator is $\frac{1}{2}\hbar\omega$.

(6 marks)

(c) An electron is moving freely inside a one-dimensional infinite potential box with walls at x = 0 and x = a. If the electron is initially in the ground state (n = 1) of the box and if the right-hand side wall is moved instantaneously from x = a to x = 4a, calculate the probability of finding the electron in:

(i) the ground state of the new box. (4 marks)

(ii) the first excited state of the new box. (3 marks)

Question 5 (20 marks)

(a) A particle of mass m is in a one-dimensional potential energy field defined by

$$V(x) = \begin{cases} \infty, \text{if} -\infty < x < 0 \\ -V_0, \text{if} 0 < x < a \\ 0, \text{if} a < x < \infty \end{cases}$$

Show that $tank_0 = -\frac{\alpha}{k_0}$ where the symbols have their usual meanings and α and k_0 have to be defined. (10 marks)

(b) The wavefunction of a particle moving in one dimension is given by

$$\Psi(x) = \sqrt{\frac{2}{a}} sin\left(\frac{n\pi x}{a}\right).$$

Calculate the expectation values of position $\langle x \rangle$ and of the momentum $\langle p_x \rangle$. (10 marks)