JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2nd YEAR 1st SEMESTER 2023/2024 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: WAB 2203

COURSE TITLE: FUNDAMENTALS OF ACTUARIAL MATHEMATICS II

EXAM VENUE: **STREAM: EDUCATION, ACTUARIAL**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1 [30 marks]

a) Suppose that u_x $is\ continuous\ on\ (\alpha, \omega), show\ that$

$$\mu_x = \frac{-l'_x}{l_x}$$ [3marks]

b) A man aged 50 has just retired because of ill health. Up to exact age 58 he will be subject to a constant force of mortality of 0.019803 p.a., after which his mortality will be that of E.L.T. No. 12 - Males. Find the probability that he will [6marks]
(i) die before age 55,
(ii) live to age 65,
(iii) die between ages 55 and 60.

c) Assuming there is Uniform Distribution of Deaths (U.D.D), show the relationship of $m_x\ and\ q_x$ [3marks]

d) Show $that\ A_x = vq_x + vp_xA_{x+1}$ [3marks]

e) (ii) Given that p_{60}= 0.985*;* p_{61} = 0.98*; i* = 0.05 and A_{62}= 0:6 , evaluate A_{61} and A_{60}[3marks]

f) Write down an expression for *ax* in terms of *v*, P_x, and a_{x+1}. [3marks]

g) Define the following terms [5marks]
   i. Whole life assurance
   ii. Stationary population
   iii. Term assurance
   iv. Net premium reserve
   v. Gross premium reserve

h) Write the present value of the benefits for an endowment life assurance payable immediately on death [2marks]

i) A level annuity of 1 pa is to be paid continuously to a 40-year-old male. On the basis of 4% pa interest and AM92 Ultimate mortality, calculate the expected present value of this annuity. [2marks]

Question 2 [20marks]

a) The staff of a large company is maintained as a stationary population by 500 new entrants each year at exact age 20. One third of those reaching age 30 leave immediately. Of the remainder, $\frac{1}{4}$ of those attaining age 60 retire immediately and the survivors retire at age 65.
The only other decrement is death.
Calculate
(i) the number of staff, [4marks]
(ii) the number of deaths in service each year. [6marks]
Basis: English Life Table No. 12 – Males

b) In a certain population, the force of mortality equals 0.025 at all ages. (We are assuming here that there is no upper limit to age.) Calculate:
(i) the probability that a new-born baby will survive to age 5 [2marks]
(ii) the probability that a life aged exactly 10 will die before age 12 [2marks]
(iii) the probability that a life aged exactly 5 will die between ages 10 and 12 [2marks]
(iv) the complete expectation of life of a new-born baby [2marks]
(v) the curtate expectation of life of a new-born baby. [2marks]

Question 3 [20marks]

a) On the basis of A1967-70 select mortality and 4% p.a. interest, calculate the mean present value of each of the following assurance benefits for a life aged 30:
(i) A whole life assurance for £10,000, payable immediately on death; [2marks]
(ii) A 20-year term assurance for£ 50,000, payable at the end of the year of death; [2marks]
(iii) A 20-year endowment assurance for £50,000, with the death benefit payable immediately on death; [3marks]
(iv) A deferred temporary assurance for £100,000, payable at the end of the year of death, if death occurs between ages 40 and 50 exactly. [3marks]

b) Using commutation functions or otherwise calculate the values of the following: [10marks]

(i) $A_{[40]:\overline{10|}}$ on A1967-70, 4% p.a. interest;
(ii) $A^{1}_{30:\overline{20|}}$ on A1967-70 Ultimate, 4%;
(iii) $\bar{A}^{1}_{30:\overline{20|}}$ on A1967-70 Ultimate, 4%;
(iv) $\bar{A}_{30:\overline{20|}}$ on A1967-70 Ultimate, 4%;
(v) $\bar{A}_{30:\overline{20|}}$ on English Life Table No.12 Males, 4%.

Question 4 [20marks]

a) Show that

i $$ {}_tV_{x:\overline{n}|} = 1 - \frac{\ddot{a}_{x+t:\overline{n-t}|}}{\ddot{a}_{x:\overline{n}|}} $$ [3marks]

ii. $$ {}_t\bar{V}(\bar{A}_{x:\overline{n}|}) = 1 - \frac{\bar{a}_{x+t:\overline{n-t}|}}{\bar{a}_{x:\overline{n}|}} $$ [3marks]

iii What is the International Notation for the reserve [2marks]

iv Express this reserve in terms of annuity functions [3marks]

v A whole life annuity is issued to a life aged x. The annuity is purchased by a single premium and a benefit of 1 is payable at the beginning of every year throughout life. Show that the net prospective and retrospective reserves are equal. [5marks]

vi. Define a reserve and explain reasons as to why hold a reserve [4marks]

Question 5 [20marks]

a) Give that $Z = g(T) = Sv^T$ is the present value of S due immediately on the death of (x), and that the mean of Z $is\ S\ \bar{A}\ x$. What is the variance of Z? [6marks]

b) Calculate the annual premium for a term assurance with a term of 10 years to a male aged 30, with a sum assured of £100,000, assuming AM92 Ultimate mortality and interest of 4% pa. Assume that the death benefit is paid at the end of the year of death. [8marks]

c) An office issues a large number of 25-year without-profit endowment assurances on lives aged exactly 40. Level annual premiums are payable throughout the term, and the sum assured of each policy is £20,000, payable at the end of the year of death or on survival to end of the term. The office's premium basis is: A1967-1970 ultimate; 4% p.a. interest; expenses are 5% of each annual premium including the first, with additional initial expenses of 1% of the sum assured. Calculate the annual premium for each policy. [6marks]