JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OFBIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF   SCIENCE ACTUARIAL
3RD YEAR 1st SEMESTER 2023/2024 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: WAB 2303

COURSE TITLE: ACTUARIAL MATHEMATICS II

EXAM VENUE: STREAM: EDUCATION, ACTUARIAL

DATE: EXAM SESSION:

TIME: 2.00 HOURS

<u>**Instructions:**</u>

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1   [30marks]

a. With reference to sickness functions, define the following: [5marks]
   - Deferred period
   - Waiting period
   - The off-period
   - $\bar{z}_x^{m/n}$
   - $\bar{z}_x^{13}$

b. A life insurance company uses the following 3-state model, to calculate premiums for a 3-year sickness policy issued to healthy policyholders age 50 exact at inception.
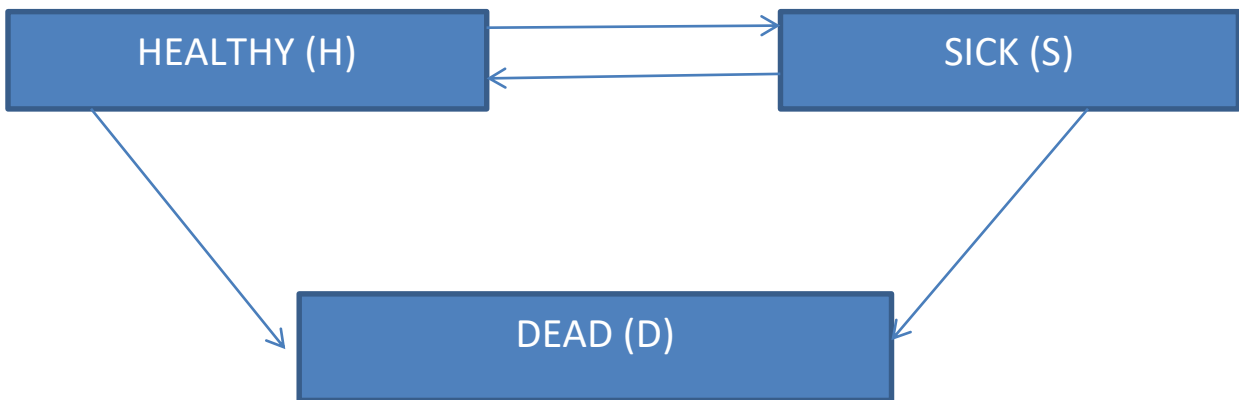
   Which in return for a single premium of P payable at the outset the company will pay a benefit of £10,000 at the end of each of the 3 years if the policyholder is sick at that time.

   Let St represent the state of the policyholder at age $50 + t$, so that $S0 = H$ and for $t = 1, 2$ and 3, $St = H$, S or D.

   The life insurance company uses transition probabilities defined as follows:

   $P_{50+t}^{ij} = P(S_{t+1} = j | S_j = i)$

   For $t = 0, 1$ and 2 the transition probabilities are:

   $$P_{50+t}^{HD} = 0.05 \qquad P_{50+t}^{SD} = 0.15 \qquad P_{50+t}^{SH} = 0.80 \qquad P_{50+t}^{HS} = 0.1$$

   The life insurance company calculates P as the expected present value of the benefit payments, assuming interest at 6% per annum and expenses of 5% of P. Find P.   [6marks]

c. A healthy life aged exactly 35 has a policy providing an income benefit of £50 per week payable during sickness. The benefit is not payable beyond age 60. There is no deferred or waiting period. Calculate the present value of this benefit.   [5marks]
   Basis: Mortality: English Life Table No. 12-Males
   Sickness: Manchester Unity Sickness Experience 1893/97
   Occupation Group AHJ
   Interest: 4% per annum

d. In the context of Manchester Unity sickness functions, state the relationship between
$S_{26}^{26/26}$ and $Z_{26}^{26/26}$ [4marks]

e. Define tP_{xy} and show that $tP_{xy} = tP_x + tP_y - tP_{xy}$ [3marks]

f. Prove that:
$$tP_{xy} = \exp(-\int_0^t \mu_{x+r:y+r} \, dr)$$ [3marks]

g. A multiple decrement table is subject to two forces of decrement, α and β. Under the assumption of a uniform distribution of the independent decrements over each year of age, $aq_x^\alpha = 0.15$ and $aq_x^\beta = 0.07$. Calculate q_x^α and q_x^β . [4marks]

Question 2   [20marks]

a) A life insurance company uses the three-state illness-death model as described above to calculate premiums for a 3-year sickness policy issued to healthy policyholders aged 60.
S_t denotes the state occupied by the policyholder at age $60 + t$, so that $S_0 = a$ and $S_t = a, i$ or d for $t = 1, 2, 3$.
The transition probabilities used by the insurer are defined in the following way:
$P_{60+t}^{jk} = P(S_{t+1} = k/S_t = j)$
For $t = 0, 1, 2$, it is assumed that:
$$P_{60+t}^{aa} = 0.9, \qquad P_{60+t}^{ai} = 0.08, \quad P_{60+t}^{ia} = 0.7, \qquad P_{60+t}^{ii} = 0.25$$
What is the probability that a new policyholder is:
(i) ill at exact age 62? [5marks]
(ii) dead at exact age 62? [5marks]

b) A population is subject to two modes of decrement, α and β . In the single decrement tables:
$$_tP_{60}^\alpha = (\frac{40-t}{40})^2 \qquad\qquad \text{for } 0 \leq t \leq 40$$
and

$$_tP_{60}^\beta = (\frac{40-t}{40})^2 \qquad\qquad \text{for } 0 \leq t \leq 40$$

Calculate the value of $(aq)_{60}^\alpha$ [10marks]

Question 3 [20marks]

a. Using Manchester Unity Sickness Tables, state the meaning of:
i.   The force of sickness $\bar{Z}_x$ [2marks]
ii.   The annual rate of sickness S_x [2marks]

b. For a certain insurance sickness policy provides combined endowment and sickness benefits, the sickness benefit is £200 per week for the first 26 weeks of sickness, £150 per week for the next 26 weeks and £100 per week thereafter while sickness lasts. All sickness payments cease on a policyholder's 65th birthday. There are no waiting or deferred periods.
The endowment part of the policy pays £10,000 immediately on the death of the policyholder or on survival to age 65.
Premiums are waived during periods of sickness.
Determine the level premium per annum payable continuously by a new policyholder aged 35.
Premiums are payable to age 65 but cease on earlier death.
Basis: Sickness: S(MU)

Mortality: ELT 15 (Males)
Interest: 4% per annum
Expenses: Nil   [16marks]

Question 4   [20marks]

a. The members of a large company's manual workforce are subject to three modes of decrement, death, withdrawal and promotion to supervisor. It is known that these workers' independent rates of mortality are the basis of A1967-70 ultimate, the independent withdrawal rate is 0.04 at each age, and their independent promotion rate is 0.02 at age 50 and 0.03 at age 51.
      i. Draw up a service table for manual workers from age 50 to age 51 with a radix of 100,000 at age 50, including the value of $(al)_{52}$ [13marks]
     ii. Calculate the probability that a life aged exactly 50 will gain promotion within 2 years.
                                                                                 [2marks]
b. Calculate the annual premium for a term assurance with a term of 10 years to a male aged 30, with a sum assured of £100,000, assuming AM92 Ultimate mortality and interest of 4% pa. Assume that the death benefit is paid at the end of the year of death.   [5marks]

Question 5 [20marks]

An endowment assurance policy, with sum assured £8000, term five years and level annual premiums, is issued to a life aged 50.
The annual premium is calculated on the following basis:

Mortality: A1967-70 ultimate
Interest: 5% per annum
Initial expenses: £200
Renewal expenses (associated with the payment of the second and each subsequent premium): £50 at the time of payment of the second premium, increasing thereafter by 10% per annum (compound).
The death benefit is payable at the end of the year of death.

(i) Determine the annual premium   [5marks]
(ii) Assume that in calculating cash flows and profit signature for the policy the office uses the premium basis.
On this basis determine for each year of the policy's duration
(a) the `in force' expected cash flow, and   [10marks]
(b) the `initial' expected cash flow.   [5marks]