JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ACTUARIAL SCIENCE

2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2109

COURSE TITLE: INTRODUCTION TO PROBABILITY THEORY

EXAM VENUE: **STREAM: ACTUARIAL SCIENCE**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One Compulsory (30mks)

a) Briefly explain the meaning of the following terms as used in Probability (8marks)

i) Equally likely events

ii) Independent events

iii) Exhaustive events

iv) Probability

b) Prove that

$if\ A\ and\ B\ are\ any\ two\ events, then\ P(A\ U\ B) = P(A) + P(B) - P(A\ n\ B)$

(5marks)

c) Suppose $f(x) = \frac{1}{2}(x-2),\ \ x = 1,2,3,4$. Is f(x) a Probability Mass Function (5marks)

d) Given $f(x) = \begin{cases} c\sqrt{x} & 0 < x < 1 \\ 0 \; otherwise \end{cases}$

Obtain i) c

ii) $P(X < \frac{1}{4})$ (5marks)

e) Let $f(x) = \begin{cases} \frac{1}{5}\,(x+3), & 0 < x < 1 \\ 0\,, otherwise \end{cases}$

Obtain i) E(X) (2marks)

ii) Var (X) (2marks)

iii) Var (3X + 3) (3marks)

Question Two (20mks)

a) A group of 50 people was asked which of the three novels they read A, B or C. the results showed that 16 read A, 25 read B, 5 read C, 14 read both A and C while 2 read all the three. If a person is chosen at random from these group, find the probability that he

i) Reads A only (3marks)

ii) Reads only one of the novels (3marks)

iii) Read at least one of the novels (4marks)

b) At a certain assembly plant, three machines make 45%, 30%, and 25%, respectively, of the products. It is known from the past experience that 2%, 3% and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.

i. What is the probability that it is defective? (5marks)

ii. If a product were chosen randomly and found to be defective, what is the probability that it was made by machine 3? (5marks)

Question Three (20mks)

a) Let X be a random variable with pdf given by $f_x(x) = \begin{cases} cx^2 & x \leq 1 \\ 0, otherwise \end{cases}$

i) Find the constant c. (4marks)

ii) Find E(X) and Var(X). (6marks)

iii) Find $P(X \geq \frac{1}{2})$ (4marks)

b) Let X be a continuous random variable with pdf

$f_X(x) = \begin{cases} 4x^3 & 0 < x \leq 1 \\ 0, otherwise \end{cases}$

Find $P(X \leq \frac{2}{3} | X > \frac{1}{3})$ (6marks)

Question Four (20mks)

Let X be a discrete random variable with the following pmf

$$P(X) = \begin{cases} 0.1 & for\ x = 0.2 \\ 0.2 & for\ x = 0.4 \\ 0.2 & for\ x = .05 \\ 0.3 & for\ x = 0.8 \\ 0.2 & for\ x = 1 \\ & otherwise \end{cases}$$

i. Find RX, the range of the random variable X. (5marks)
ii. Find P(X≤0.5). (5marks)
iii. Find P(0.25<X<0.75) (5marks)
iv. Find P(X=0.2|X<0.6). (5marks)

Question Five (20mks)

a) Customers arrive in a bank according to a Poisson process with rate λ = 5 per hour. Given that the store opens at 9:00am,

i. what is the probability that exactly one customer has arrived by 9:30? (4mks)
ii. what is the probability that five have arrived by 11:30? (4mks)
iii. given 1, what is the probability that total of five have arrived by 11:30? (6mks)
iv. given 1 and 2, what is the probability that the total of 10 has arrived by the time the store closes (5:00pm)? (6mks)

b) You take an exam that contains 20 multiple-choice questions. Each question has 4 possible options. You know the answer to 10 questions, but you have no idea about the other 10 questions so you choose answers randomly. Your score X on the exam is the total number of correct answers.

i)Find the PMF of X. (10marks)