



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE**

**ACTUARIAL**

**3<sup>RD</sup> YEAR 1<sup>ST</sup> SEMESTER 2023/2024**

**REGULAR (MAIN)**

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**COURSE CODE: WAB 2301**

**COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATIONS I**

**EXAM VENUE:**

**STREAM: (BSc Actuarial Science)**

**DATE:**

**EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

## QUESTION ONE [ 30 marks]

- a. State the three theories explaining the shape of the yield curve.  
(6 marks)
- b. Describe a one-factor term structure model?  
(2 marks)
- c. A three-month forward contract exists on a zero-coupon corporate bond with a current price per £100 nominal of £42.60. The yield available on three-month government securities is 6% *pa* effective.  
Calculate the forward price.  
(2 marks)
- d. The current price of a share is £200. The share pays dividends continuously to provide a fixed dividend yield, and the current dividend is £5 *pa*.  
Calculate the forward price of a five-year contract on one share if the risk-free force of interest is 5% *pa*.  
(3 marks)
- e. Suppose that the exercise price of a 3-month European call option on Share X is 100 and the continuously compounded risk-free rate of return is 12% *pa*.  
Calculate the lower bound for the option's price if the current price of Share X is:  
(i) 115  
(ii) 125.  
(4 marks)
- f. Calculate the lower bound for a 3-month European put option on Share X if the current share price is 95, the exercise price is 100 and the continuously compounded risk-free rate is 12% *pa*.  
(2 marks)
- g. A man now aged exactly 50 has built up a savings fund of £400,000. In order to retire at age 60, he will require a fund of at least £600,000 at that time. The annual returns on the fund,  $i$ , are independent and identically distributed, with  $1+i \sim \log N(0.075, 0.1^2)$ .  
Calculate the probability that, if the man makes no further contributions to the fund, he will be to retire at age 60.

(5 marks)

- h. When valuing derivatives it is often assumed that the price of the underlying security follows a geometric Brownian motion with stochastic differential equation:

$$dS_t = S_t(\mu dt + \sigma dZ_t)$$

where  $Z_t$  represents a standard Brownian motion. List the 2 advantages and 2 disadvantages of this assumption.

(4 marks)

- i. Explain what is meant by the continuous-time lognormal model of security prices.

(2 marks)

## QUESTION TWO [ 20 marks]

- a. The short rate of interest is governed by the stochastic differential equation (SDE):

$$dr_t = 0.6(0.04 - r_t)dt + 0.006dB_t$$

where  $B_t$  is a standard Brownian motion.

By considering the function  $f(r_t, t) = r_t e^{0.6t}$ , or otherwise, solve this SDE.

(10 marks)

- b. A bond trader assumes that  $f(t, T)$ , the instantaneous forward rate of interest at time  $T$  implied by the market prices of bonds at the current time  $t$ , can be modelled by:

$$f(t, T) = 0.04e^{-0.2\tau} + 0.06(1 - e^{-0.2\tau}) + (1 - e^{-0.2\tau})e^{-0.2\tau}$$

where  $\tau = T - t$

- i. Sketch a graph of  $f(t, T)$  as a function of  $\tau$ .

(3 marks)

- ii. Calculate the following quantities using this model:

- the instantaneous forward rate of interest in two years' time
- the current price of a 10-year zero-coupon bond

c) the current 10-year spot rate.

You should express your answers to (a) and (c) as annualized continuously compounded rates.

(7 marks)

### QUESTION THREE [ 20 marks]

a. A fixed-interest security pays coupons of 8% *pa* half-yearly in arrears and is redeemable at 110%.

Two months before the next coupon is due, an investor negotiates a forward contract to buy £60,000 nominal of the security in six months' time. The current price of the security is £80.40 per £100 nominal and the risk-free force of interest is 5% *pa*.

i. Calculate the forward price.

(2 marks)

ii. On the same day, a different investor negotiates a forward contract to purchase £50,000 nominal of the security in ten months' time.

Calculate the forward price of this contract.

(4 marks)

b. The annual returns on a fund,  $i$ , are independent and identically distributed. Each year, the distribution of  $1+i$  is lognormal with parameters  $\mu = 0.075$  and  $\sigma^2 = 0.0252$ .

Calculate the upper and lower quartiles for the accumulated value at the end of 5 years of an initial investment of £1,000.

(4 marks)

c. A lump sum of \$14,000 will be invested at time 0 for 4 years at an annual rate of return,  $i'$ . The rate of return, once determined, will be the same in each of the four years.  $1+i'$  has a lognormal distribution with mean 1.05 and variance 0.007.

Calculate the probability that the investment will accumulate to more than \$20,000 in 4 years' time.

(10 marks)

## QUESTION FOUR [ 20 marks]

- a. The returns from an investment are assumed to conform to the fixed rate model with the distribution of rates as specified below:

$$i_k = \begin{cases} 0.06 & \text{with probability } 0.2 \\ 0.08 & \text{with probability } 0.7 \\ 0.10 & \text{with probability } 0.1 \end{cases}$$

- i. Calculate the expected accumulated value at the end of 5 years of an initial investment of £5,000. (3 marks)
- ii. Calculate the accumulated value at the mean rate of return. (2 marks)
- iii. Calculate the variance of the accumulated value of the investment (5 marks)
- b. Calculate the mean and variance of the accumulated value at the end of 25 years of an initial investment of £40,000, if the annual rate of return in year  $k$  is independent of that in any other year and  $i_k \sim \text{Gamma}(16, 200)$  for all  $k$ . (10 marks)

## QUESTION FIVE [20 marks]

- a. The shares of Abingdon Life can be modelled using a lognormal model in which the drift parameter,  $\mu=0.104$  pa and volatility,  $\sigma=0.40$  pa. If the current share price is 2.00, derive a 95% confidence interval for the share price in one week's time, assuming that there are exactly 52 weeks in a year. (10marks)
- b. An investor has decided to model PPB plc shares using the continuous-time lognormal model. Using historical data, the investor has estimated the annual drift and volatility parameters to be 6% and 25% respectively. PPB's current share price is \$2.
- i. Calculate the mean and variance of PPB's share price in one year's time. (5marks)
- ii. Calculate the probability that PPB's shares fall in value over the next year. (5marks)