JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OFBIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR 1st SEMESTER 2023/2024 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: WAB 2305

COURSE TITLE: PENSION MATHEMATICS

EXAM VENUE: **STREAM: EDUCATION, ACTUARIAL**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One [30 marks]

a) Contributions to a pension scheme by employees are made at a rate of 5% of salary when aged under 35, 6% between ages 35 and 45, and $7\,{}^{1}/_{2}$ % when aged 45 or over. Calculate the present value of the future contributions payable by a member aged exactly 30 who in the past year have received a total salary of £12,718. **[4marks]**

b) Show that

$\bar{R}_x^{ia} = \sum_{t=0}^{64-x}(t + {}^{1}/_{2})C_{x+t}^{ia}$ **[5marks]**

c) Differentiate between defined benefit schemes and defined contribution scheme[4marks]

d) If a life aged 35 contributes £500 each year to his pension scheme, calculate the value of his future contributions. **[3marks]**

e) A pension scheme provides each member who retires (whether for age" or ill- health" reasons) with an annual pension of $\frac{1}{60}th$ of his average annual income over a member's entire service, for each year of service. Fractions of years of service are included when calculating the amount of pension payable. If contributions are paid entirely by the employer, calculate the appropriate contribution rate (as a percentage of salary) for a new entrant aged 20. **[5marks]**

f) A company has a pension scheme which provides a pension upon retirement of $\frac{1}{80}th$ of the final salary per year of service. In addition, the sum of £10,000 is paid on death in service of a member. If all contributions are paid by the employer, find the contribution rate as a percentage of salary required for a new entrant aged 40 with a salary rate of £10,000. Use the basis of Formulae and Tables" (with the supplement); final salary is the average annual salary in the 3 years prior to retirement. **[5marks]**

g) A pension scheme provides a pension of 1/60 of career average salary in respect of each full year of service, on age retirement between the ages of 60 and 65. A proportionate amount is provided in respect of an incomplete year of service. At the valuation date of the scheme, a new member aged exactly 40 has an annual rate of salary of £40,000. Calculate the expected present value of the future service pension on age retirement in respect of this member, using the Pension Fund Tables in the Formulae and Tables for Actuarial Examinations. **[4marks]**

h) Write down an expression, using commutation functions, for the EPV of a lump sum of £150,000 paid immediately on the event of ill-health retirement, for an active pension scheme member aged exactly 48. Assume that ill-health retirement is only permitted before the member reaches his 60th birthday. Define all the symbols that you use. **[4marks]**

Question Two [20marks]

A company pension scheme provides the following benefits for all members:
(1) a pension on retirement (on grounds of ill-health or of age) of one-eightieth of final pensionable salary for each year of service (including fractions),
(2) a lump sum on retirement of 3 times the annual pension,
(3) on death in service, a lump sum of Ksh. 3,000,000,
(4) on withdrawal from service, a return of the employee's contributions, accumulated at 3% per annum compound.
Final pensionable salary is defined as the average annual salary in the three years immediately before retirement. All members who reach age 65 retire immediately.

Employees contribute to the scheme at the rate of 3% of salary, payable continuously. Salaries are revised continuously. The employer's contribution rate is assessed for each member separately, and is such that the prospective reserve for each new entrant is zero. Expenses are ignored.

(a) (i) Derive a formula, in terms of suitable commutation functions, for valuing benefit (1) above in respect of a new entrant aged *x* with annual salary rate SAL. (You need not define the service table functions.) **[3marks]**

(ii) In respect of a new entrant aged *x* with annual salary rate SAL, give formulae for valuing benefits (2), (3) and (4) above, using suitable commutation functions. (You need not derive the formulae.) **[3marks]**

(iii) Hence find a formula for the employer's contribution rate for a new member aged *x* and a starting salary rate of £10,000 p.a. **[3marks]**

b) (i) Using the basis given in the pension fund section of the Formulae and Tables (and the supplement), find the value of each of the benefits (1), (2), (3) and (4) for a new entrant aged 45 with salary rate £10,000 per annum. **[8marks]**

(ii) Hence or otherwise find the employer's contribution rate for this new member. **[3marks]**

Question Three [20marks]

a) Three members of a pension scheme whose age nearest birthday is 45 have the following annual rates of salary and exact numbers of years of past service:
   A : Ksh.1,500,000 20 years
   B : Ksh.1,200,000 10 years
   C : Ksh.1,400,000 5 years

For these members find the present value of a pension, payable on age-retirement or on ill-health retirement, of $\frac{1}{100}th$ of the average salary in the final 3 years before retirement for each year of service, including fractions. Give separately the values of the past-service and future-service benefits. **[10marks]**

Derive the commutation functions for the following:

i) The value of the Past Service Pension (P.S.P.) **[5marks]**

ii) The value of the Future Service Pension (F.S.P.) **[5marks]**

Question Four [20marks]

a) A pension scheme provides a pension on age retirement of $\frac{1}{60}th$ of final pensionable salary for each year of service, with part years counting proportionately. Final pensionable salary is defined to be the average salary over the 3 years prior to retirement. Members contribute 6% of their salaries to the pension fund.

One member aged exactly 50 has 18 years of past service and earned £45,000 in the last year. Using the Pension Scheme Tables from the Actuarial Formulae and Tables, calculate:

(i) the expected present value of this member's past service benefit **[5marks]**

(ii) the expected present value of this member's future service benefit **[4marks]**

(iii) the expected present value of this member's future contributions. **[4marks]**

b. The mean present value of the future contributions (of employee, employer or both) at rate *k*% of salary for a member age *x* with current salary rate of £SAL per annum is

$$\frac{k}{100} \cdot \frac{SAL}{\bar{s}_x} \int_0^{65-x} v^t \frac{l_{x+t}}{l_x} \bar{s}_{x+t} \, dt$$

Show that the commutation function is given by **[7marks]**

$$\frac{K}{100} \frac{SAL}{\bar{s}_x D_x} \cdot {}^{s}\overline{N}_x$$

Question Five [20marks]

a. A pension scheme provides the following benefit to the spouse of a member, following the death of the member in retirement: A pension of £10,000 per annum payable during the lifetime of the spouse, but ceasing 30 years after the death of the member if that is earlier. All payments are made on the anniversary of the member's retirement. Calculate the expected present value of the spouse's benefit in the case of a female member retiring now on her 60th birthday, who has a husband aged exactly 64. Basis: a(55) Ultimate mortality at 8% per annum interest. **[10marks]**

b. A retiring employee aged exactly 60 is given a choice between the following two pensions: Pension A is payable annually in arrear throughout the pensioners lifetime, with at least 4 payments guaranteed to be made. The first payment is £20,000 and payments increase by 0.9709% per annum compound thereafter. Pension B is payable annually in arrear, with an initial payment of £13,000. Each subsequent payment increases by £1,000 and payments cease immediately on death. Calculate the expected present value of each pension using the following basis:
Mortality: A1967-70 Select
Interest: 4% per annum **[10marks]**