JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH IT

3rd YEAR 1st SEMESTER 2023/2024 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: WAB2309

COURSE TITLE: THEORY OF ESTIMATION

EXAM VENUE: **STREAM: (BSc. Actuarial Science)**

DATE: **EXAM SESSION: Sep-Dec 2023**

TIME: 2.00 HOURS

Instructions:

i. Answer questions one and any other two.
ii. Candidates are advised not to write on the question paper.
iii. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE (30 marks)

a) State and explain four important criteria of a good estimator (4 marks)

b) Let $x_1, x_2, \cdots, x_n$ be a random sample of size n from a Poisson population with parameter λ. Find unbiased estimators of λ and λ^2. Also, find the variance of the estimator of λ. (4 marks)

c) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations selected from normal population with mean μ and variance σ^2. Find unbiased estimators of μ and σ^2. Also, find the variance of the estimators. (4 marks)

d) Let $x_1, x_2, \cdots, x_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Consider two estimators of μ as $T_1 = \frac{1}{n}\sum_1^n x$ and $T_2 = \frac{1}{n+1}\sum_1^n x$

Find relative efficiency of T_2 compared to T_1 (4 marks)

e) Let $x_1, x_2, \cdots, x_n$ be a sample of n observations from a population having pdf

$$f(x) = \begin{cases} \frac{2(\theta - x)}{\theta^2}, & 0 \le x \le \theta \\ 0, & \text{Otherwise} \end{cases}$$

Show that $T_n = 3\bar{x}$ is a mean square consistent estimator of θ (4 marks)

f) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from a population having pdf

$$f(x) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{Otherwise} \end{cases}$$

Show that $\left(\frac{n+1}{n}\right) x_{(n)}$ is the most efficient estimator of θ,

Where: $x_{(n)} = \max(x_1, x_2, \cdots, x_n)$ (4 marks)

g) Let $x_1, x_2, \cdots, x_n$ be a random sample of size n observations from a population having pdf

$$f(x, \theta) = \theta e^{-\theta x}, \quad x > 0$$

The "apriori" distribution of Q is

$$f_1(\theta) = \frac{1}{\theta}, \quad 0 < x < \infty$$

Find Bayes estimator of θ (3 marks)

h) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from a population having pdf

$$f(x, \theta) = \theta e^{-\theta x}, \quad 0 < x < \infty$$

Find the estimators of θ, θ^2 and θ^3 by method of moment. (3 marks)

QUESTION TWO (20mks)

a) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from a population whose pdf is

$$f(x) = \frac{1}{b - a}, \quad a < x < b$$

Find the estimators of a and b by method of moment. (5 marks)

b) Let $x_1, x_2, \cdots, x_m$ be a random sample of m observations from binomial population with parameters n and p. Find the estimators of n of n and p by method of moment. (5 marks)

c) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from a population having pdf.

$$f(x) = \frac{\beta^\alpha}{\Gamma\alpha} x^{\alpha-1} e^{-\beta x};\ 0 \le x \le \infty;\ \alpha > 0;\ \beta > 0$$

Find the estimator of α and β by method of moment. (10 marks)

QUESTION THREE (20mks)

a) The following data represent the body weight (y kg), body length (x_1, cm) of 12 randomly selected sea fish.

x	12	20	14	25	18	16	10	18	18	20	16	12
y	0.5	0.8	0.7	2.0	1.2	0.9	0.4	0.9	1.4	1.5	0.8	0.6

Assume the linear model of y, x as $y = \beta_0 + \beta_1 x + \varepsilon$

Estimate the parameters β_0 and β_1 by method of least squares. (20 marks)

QUESTION FOUR (20mks)

a) Let $x_1, x_{2,} \cdots, x_n$ be a random sample of n observations from a Poisson population with parameter θ such that (i) $f_1(\theta) = e^{-\theta}, \theta > 0$ (ii) $f_1(\theta) = \frac{1}{\alpha} e^{-\theta/\alpha}, \theta > 0$. Using quadratic loss function find Bayes estimator of θ. (15 marks)

b) Let $x_1, x_{2,} \cdots, x_n$ be a random sample of n observations from a population with probability function $P(x) = \frac{1}{N}$, x=1,2, …, N

Find ML estimator of N (5 marks)

QUESTION FIVE (20mks)

a) Let $x_1, x_{2,} \cdots, x_n$ be a random sample of n observations from a Poisson population with parameter λ. .

Find a sufficient statistic for λ (8 marks)

b) Let $x_1, x_{2,} \cdots, x_n$ be a random sample of n observations from a population having pdf.

$$f(x) = \frac{1}{\beta^{\alpha} \Gamma\alpha} x^{\alpha-1} e^{\frac{-x}{\beta}};\ 0 \le x \le \infty;\ \alpha > 0;\ \beta > 0$$

Find joint complete sufficient statistic of α and β (12 marks)