JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCE UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL
EXAMINATION 2023/2024
REGULAR (MAIN)

COURSE CODE: WAB 2417
COURSE TITLE: NON PARAMETRIC METHODS

EXAM VENUE:
DATE:
TIME: 2.00 HOURS

## Instructions:

1. Answer question 1 (Compulsory) and ANY other 2 questions
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE (20 MARKS)
a) A continuous random variable X has probability density function defined by

$$
f(x)=\left\{\begin{array}{c}
\frac{1+x}{6}, 0<x<3 \\
0 \text { otherwise }
\end{array}\right.
$$

Obtain the Median and the $80^{\text {th }}$ percentile for this distribution
[ 6 Marks]
b) The masses of goods earmarked for consignment were measured and recorded in kilograms as follows:
$34,35,27,26,29,30,25,36,33,20,18,23,27,22,19,23,29,33,29,24,26$
Use the one sample sign test, to check at $5 \%$ level of significance the claim that the Median weight is less than 23 Kilograms?
[6 Marks]
c) The ages of patients were recorded as they entered a clinic on one particular day. The recording was done according to the time that each patient arrived at the clinic. One thinks that there is a pattern associated with the age of the patient and the time they appeared at the clinic.Use Cox Stuart test at 5\% level of significance to test appropriate hypothesis on the claim made. Use the data set below.

$$
34,45,23,78,45,17,56,56,56,78,45,43,34,35,46,44,29,67,54,31,46,87 \text {, }
$$

57, $\quad 54,63,71,77,76,78,88,67,69,65,86,76,76,75,83,84$.
Marks]
d) Let $Y_{1}<Y_{2}<Y_{3}<\cdots<Y_{8}$ be the order statistic of a random sample from a distribution of the continuous type. Compute the value of $\operatorname{Pr}\left(Y_{2}<\xi_{0.4}<Y_{6}\right)$ hence state the associated confidence interval.
Marks]
e) In a simulation study the following four digit numbers were generated from a computer package

$$
\begin{array}{llllll}
3034 & 4991 & 5220 & 6767 & 8125 & 2024 \\
6054 & 2178 & 3002 & 5686 & 7354 & 9701
\end{array}
$$

Are these numbers random?
Marks]

## QUESTION TWO (20 MARKS)

a) Eleven companies did a survey on customer satisfaction based on the products that they sell. Customers were to award scores in a scale of 20 to 50 on their levels of satisfaction. The average scores by Male and female customers for the companies were follows;

| Company | A | B | C | D | E | F | G | H | I | J | K |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Rank by <br> Males | 28 | 35 | 24 | 27 | 31 | 39 | 42 | 41 | 36 | 37 | 34 |
| Rank by <br> Females | 24 | 32 | 28 | 29 | 30 | 35 | 41 | 43 | 40 | 33 | 37 |

One claims that the level of satisfaction by gender in not different. Compute and test Kendall's rank correlation coefficient at 5\% level of significance and interpret it [10Marks]
b) Two samples A and B were recorded as follows.

| A | 51 | 56 | 58 | 46 | 60 | 55 | 59 | 63 | 56 | 54 | 57 | 61 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| B | 50 | 53 | 49 | 56 | 59 | 52 | 47 | 50 | 62 | 64 |  |  |

An One claims that the samples were drawn from the same population. By stating appropriate hypothesis,test this claim at $5 \%$ level of significance. Use KolmogorovSmirnorv test.
Marks]

## QUESTION THREE (20 MARKS)

a) Some dairy cattle are fed on supplements to try and boost their milk production. The average amount of milk per day for 8 animals was recorded in the first week before supplements and in the second week after supplements were administered. The data set was as follows

| First <br> week | 10 | 11 | 15 | 16 | 12 | 13 | 11 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Second <br> week | 12 | 11 | 17 | 18 | 11 | 15 | 12 | 11 |

Do you think that supplements were effective? Use Wilcoxon signed rank test in making an appropriate decision. State appropriate hypothesis.
Marks]
b) Let $Y_{1}<Y_{2}<Y_{3}<\cdots<Y_{100}$ be the order statistics of a random sample of size 100 from a distribution of the continuous type. Find i and j where $\mathrm{i}<\mathrm{j}$ such that

$$
\operatorname{Pr}\left(Y_{i}<\xi_{0.6}<Y_{j}\right)=0.95
$$

Marks]

## QUESTION FOUR (20 MARKS)

a) Four vaccines were tested on 20 rats. Each of the four vaccines was given to 5 rats. The rats are assumed to be about the same age and are of the same breed. The time taken to develop full immunity by the rats in days was recorded as follows:

|  | observation |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Vaccines |  |  |  |  |  |  |
| 1 | 25 | 24 | 26 | 27 | 23 |  |
| 2 | 29 | 19 | 27 | 24 | 18 |  |
| 3 | 22 | 24 | 20 | 21 | 26 |  |
| 4 | 20 | 18 | 24 | 26 | 20 |  |

Stating appropriate hypothesis, use Kruskall Wallis Non-Parametric test to check for similarity of the treatments.
b) A group of 21 invalids were treated for fever using a paracetamol. The signs of the differences of their body temperature in comparison with the previous were recorded as follows:

$$
+++++--+++++++--++
$$

One suspects that the paracetamol was effective in managing body fever. By stating clearly the null and alternative hypotheses, apply the two sample sign test to test the hypothesis at $5 \%$ level of significance.
[7 Marks]

## QUESTION FIVE (20 MARKS)

a) In a study observations were made as follows
$30,34,49,91,52,20,67,67,81,25,20,24,90,60,54,21,78$. Based on the order statistics $Y_{i}=21$ and some $Y_{j}$ to be identified from the sample;
i. Obtain the confidence coefficient for the quartile of order $\mathrm{p}=0.5$

Marks]
ii. Obtain the associated confidence interval.
iii. At what level of significance would one test hypothesis based on ii above?
b) The age at which one first used contact lenses were recorded against the gender male or female. The number of males and females in the various age bands were as follows;

| Age when <br> contact lenses <br> were first used | Gender |  |
| :--- | :--- | :--- |
|  | male | Female |
| Below 15 | 2 | 8 |
| $15-19$ | 38 | 93 |
| 20 and above | 22 | 15 |

Do you think there is association between gender and age at which lenses were first used. Test this at $5 \%$ level of significance.

