JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR BACHELOR OF ACTUARIAL SCIENCE 2023/24

MAIN REGULAR

COURSE CODE: WAB 2102

COURSE TITLE: Fundamental of Actuarial Mathematic I

EXAM VENUE **STREAM: B.Sc. Actuarial Science Year One**

DATE:…………… **EXAM SESSION: ONE**

TIME: 2 HOURS

Instructions to the Candidate:

1. **Answer ALL in-Section A and any other two questions only in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

SECTION A

QUESTION ONE: [30 MARKS]

a) i. Describe what is represented by each of the central rate of mortality m_x and the initial rate of mortality q_x . [2 Marks]

(ii) State the circumstance in which $m_x = q_x$ [2 Marks]

b) Let Tx be a random variable denoting future lifetime after age x , and let T be another random variable denoting the lifetime of a new-born person.

(i) (a) Define, in terms of probabilities, $S_x(t)$ which represents the survival function of T_x . [1Mark]

(b) Derive an expression relating $S_x(t)$ to $S(t)$, the survival function of T . [2 Marks]

(ii) (ii) Define, in terms of probabilities involving T_x , the force of mortality, μ_{x+t}. [2Marks]

c) State the principle of correspondence in the context of a mortality investigation. [2 Marks]

d) Write down the information required to compute the exact exposed to risk in an investigation of mortality. [3 Marks]

e) A study was conducted into the mortality of persons aged between exact ages 85 and 86 years. The study took place from 1 April 2015 to 31 March 2016. The following table shows information on 10 lives observed in the study.

Life Number	Date of 85th Birthday	Date of death
1	1 August 2014	-
2	1 November 2014	-
3	1 January2015	1 February 2016
4	1 February 2015	-
5	1 March 2015	-
6	1 April2015	1 January n2016
7	1 June 2015	1 November 2015

8	1 July 2015	-
9	1 September 2015	1 March 2016
10	1 January 2016	-

i. Calculate a central exposed to risk for the 10 lives in the sample, working in months.

[3 Marks]

ii. Give the maximum likelihood estimate of the mortality hazard at age 85 last birthday. [2 Marks]

iii. Estimate q85 [3 Marks]

f) Suppose $\mu_{x+t} = t$ = t for $t \geq 0$. Calculate;

i. $tP_x \; \mu_{x+t}$ [2 Marks]

ii. $\mathring{e}_x$

[2 Marks]

g) Using ELT males' mortality, approximate the following:

i. $20P_{30}$ [2 Marks]

ii. Curtate expectation of life for 21-year-old actuarial student [2 Marks]

SECTION B: [40 MARKS]

QUESTION TWO: [20 MARKS]

(i) Define the force of mortality m_x of a random variable T denoting length of life.

[2 Marks]

The mortality of a certain species of animal has been studied. It is known that at ages under five years the force of mortality m is constant.

(ii) Write down an expression, in terms of μ for the probability that an animal will survive from birth to exact age five years. [4 Marks]

Mortality of these animals at ages over five years exact is incompletely understood.

However, it is known that the probability that an animal aged exactly five years will survive until exact age 10 years is twice the probability that an animal aged exactly five years will survive until exact age 20 years.

Assume that the force of mortality θ is constant at ages over five years exact.

(iii) Calculate θ [4 Marks]]

(iv) Calculate the expectation of life at birth for these animals if $\theta = \mu$. [4 Marks]

(v) Derive an expression, in terms only of μ, for the expectation of life at birth for these animals if $\theta \neq \mu$. [6 Marks]

QUESTION THREE: [20 MARKS]

(a) Suppose that the lifetime Y of a newborn is exponentially distributed with mean 75.

i. $S_Y(t)$ [2 Marks]

ii. Probability that a newborn is still alive at age 100 [3 Marks]

iii. Probability that newborn dies between ages 60 and 75 [3 Marks]

iv. Calculate the force of mortality [3 Marks]

(b). The distribution function T_x is $F_x(t)$. Derive the probability density function (pdf) of this live distribution denoted $f_x(t)$. [9 Marks]

QUESTION FOUR: [20 MARKS]

(a) Show that $tP_x = e^{\int_x^{x+t} \mu_s ds}$ [10 Marks]

(b) i. Differentiate between initial and central exposed to risk [4 Marks]

ii. Explain three methods of Calculating central exposed to risk [6 Marks]

QUESTION FIVE: [20 MARKS]

4. (a). If $\mu_x = 0.01908 + 0.001(x - 70)$ for $x \geq 55$, calculate the probability of a person aged 60 dying in the next 5 years [6 Marks] .

(b). In a certain population, the force of mortality (μ_x) is given as;

	μ_x
$60 \leq x \leq 70$	0.01

$70 \leq x \leq 80$	0.015
x>80	0.025

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83

[6 Marks]

(c). Using the survival function;

$$S(x) = 1 - \frac{x^2}{100} \quad \text{for } 0 \leq x < 10$$

i. Calculate probability of a person aged 2 surviving for the next 1 years

[3 Marks]

ii. Find $P[K(4) = 1]$ [3 Marks]

iii. Find the distribution function of X [2 Marks]