JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ACTUARIAL SCIENCE WITH IT

1ST YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2108

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Given that X and Y are independent random variables with probability distribution functions in the form given by $f(y) = \begin{cases} 2(1-y) & 0 \le y \le 1 \\ 0 & otherwise \end{cases}$. Determine the value of $P(X + Y \le 1)$ (5 Marks)

b) Given that the joint probability distribution function of two random variables M and N is $P(M = m, N = n) = \begin{cases} \frac{m}{35 \times 2^{n-2}} & m = 1,2,3,4; n = 1,2,3 \\ 0 & elsewhere \end{cases}$

   i. Obtain the probability function for marginal distribution of M. (2 Marks)

   ii. Find the conditional probability function of M given $N = n$. (2 Marks)

c) Suppose that the joint probability distribution function of X and Y is

$$f(x,y) = \begin{cases} \frac{3}{16}(4 - 2x - y) & x \ge 0; y \ge 0 \; and \; 2x + y \le 4 \\ 0 & otherwise \end{cases}$$

   i. Determine the conditional probability distribution of Y given X. (3 Marks)

   ii. Obtain the $P(X \ge 2 | Y = 0.5)$ (2 Marks)

d) Given a random variable X with probability distribution function given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \lambda > 0; x > 0 \\ 0 & otherwise \end{cases}$$

Obtain;

i. $E(X)$ (4 Marks)

ii. $var(X)$ (4 Marks)

e) Suppose X and Y have a continuous joint distribution for which the joint probability distribution function is defined as follows:

$$f(x,y) = \begin{cases} \frac{3}{2}(y^2) & 0 \le x \le 2; 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

Determine whether the results $P(X < 1)$ and $P\left(Y \ge \frac{1}{2}\right)$ are independent? (8 Marks)

QUESTION TWO (20 MARKS)

Suppose that X and Y have a discrete joint probability distribution function defined as $f(x,y) = \begin{cases} \frac{1}{30}(x+y) & x = 0,1,2 \, ; y = 0,1,2,3 \\ 0 & otherwise \end{cases}$.

a) Determine the marginal probability function of X and Y, then represent the results in tabular form. (14 Marks)

b) Obtain E(Y|X=2) (6 Marks)

QUESTION THREE (20 MARKS)

If X and Y are random variables whose joint probability distribution function is given by $f(x,y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & otherwise \end{cases}$

a) Obtain the marginal densities of X and Y (4 Marks)

b) Determine $E(X|Y)$ and $var(X|Y)$ for $0 < y < 1$ (8 Marks)

c) Determine $E(Y|X)$ and $var(Y|X)$ for $0 < x < 1$ (8 Marks)

QUESTION FOUR (20 MARKS)

a) Consider a bivariate function given by

$$f(x,y) = \begin{cases} k(6-x-y) & 0 \le x \le 2; 2 \le y \le 4 \\ 0 & otherwise \end{cases}$$

i. Determine k so that $f(x,y)$ is a joint probability distribution function of X and Y. (4 Marks)

ii. Obtain $P(X < 1, Y < 3)$ for equation (i.) above. (6 Marks)

b) Suppose that X and Y are discrete random variables with joint probability distribution functions given by the following table

		Y			$f_1(x)$
		0	1	2	
X	0	0.10	0.10	0.20	0.40
	1	0	0.15	0.05	0.20
	3	0.10	0.20	0.10	0.40
$f_1(y)$		0.20	0.45	0.35	1

Compute the coefficient of correlation between X and Y (10 Marks)

QUESTION FIVE (20 MARKS)

a) Given a random variable X with probability density function given as

$$f(x) = \begin{cases} \frac{\lambda}{\Gamma(r)}(\lambda x)^{r-1}e^{-\lambda x} & r, \lambda > 0; x \ge 0 \\ 0 & otherwise \end{cases}$$

Obtain;

i. E(X) (4 Marks)

ii. var(X) (6 Marks)

b) Given that X is a continuous random variable, then X is said to have a chi – square distribution with probability density function given by

$$f(x) = \begin{cases} \frac{1}{\Gamma\left(\frac{n}{2}\right)2^{\frac{n}{2}}} x^{\frac{n}{2}-1} e^{-\frac{x}{2}} & x > 0 \\ 0 & elsewhere \end{cases}$$

Find the moment generating function of the chi – square (10 Marks)