JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR BACHELOR OF ACTUARIAL SCIENCE 2023/24

MAIN REGULAR

COURSE CODE: WAB 2201

COURSE TITLE: Risk Theory

EXAM VENUE **STREAM: B.Sc. Actuarial Science**

DATE:…………… **EXAM SESSION: ONE**

TIME: $1^1/_2$ HOURS

Instructions to the Candidate:

1. **Answer ALL in section A and any other two questions only in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

SECTION A: [30 Marks]

a. State four properties of Moment Generating Functions (MGF). [4 Marks]

b. Let a and b be constants and let the moment generating function of X be $M_X(t)$. Find the moment generating function of the random variable $Z = a - bx$. [4 Marks]

c. The moment generating function of a random variable Y is given by;

$M_Y(t) = (1-4t)^{-2} \ for \ t < 0.25$.

Calculate $E(Y)$ and $Var(Y)$ [6 Marks]

d. If $X \sim Poisson(\theta)$ and $Y \sim Poisson(\mu)$ are independent, find the probability function of $Z = X + Y$ using convolution. [4 Marks]

e. An insurance policy produces N claims where N is the random variable defined as;

$$N = \begin{cases} 0 \ with \ probability \ 0.5 \\ 1 \ with \ probability \ 0.4 \\ 3 \ with \ probability \ 0.1 \end{cases}$$

The individual claim X follows the probability distribution given by;

$$X = \begin{cases} 1 \ with \ probability \ 0.9 \\ 10 \ with \ probability \ 0.1 \end{cases}$$

It is assumed that the individual claim amount X and N are independent random variables. If Z is the aggregate claim amount.

Calculate:

i. The mean of $Z, E(Z)$ [4 Marks]

ii. The variance of $Z, Var(Z)$ [4 Marks]

f. An insurance company believes that an individual claim for motor insurance in 2025 will have a mean size of 7500. Estimate the proportion of the claim that will exceed 2500 assuming that individual claim sizes follow lognormal distribution. [4 Marks]

SECTION B: [40 MARKS]

1. (a)
   i. Define the Moment generating function and state its three uses. [5 Marsk]
   ii. If the moment generating function of a function X is $M_X(t)$, then derive the expression for the moment generating function of $2x + 3$ in terms of $M_X(t)$. [4 Marks]
   iii. Hence if X is normally distributed with mean μ and variance σ^2, derive the distribution of $2x + 3$ [5 Marks]

   (b) A loss amount random variable has the moment generating function;

   $$M(t) = 0.4(1 - 20t)^{-2} + 0.6(1 - 30t)^{-3}$$

   Calculate the expected loss amount. [6 Marks]

2. Let N be the number of claims on a risk in a year. Suppose the claims $X_1, X_2, \ldots$ are independent and identically distributed random variables independent of N. Let S be the total amount claimed in one year.
   i. Derive E(S) and Var(S) [8 Marks]
   ii. Derive and expression for moment generating function of X in terms of $M_X(t)$ and $M_N(t)$ of X_1 and N respectively. [6 Marks]
   iii. If N has a Poisson distribution, find the moment generating function of S in terms of θ. [6 Marks]

3. (a) Let N be a geometric random variable with probability mass function as;

   $$P(x) = pq^x \qquad n = 0,1,2, \ldots \text{ where } 0 < q < 1.$$

   Obtain the moment generating function of $S = X_1 + X_2 + \cdots + X_N$ in terms of MGF of X, $M_X(t)$ when N is independent of $X_{i's}$ [8 Marks]

(b) Let the number of claims N be geometric random variable with

$$P(x) = pq^x \qquad n = 0,1,2, \ldots$$ and the common distribution function of X

X is;

$F(x) = 1 - e^{-x}$

That is individual claim amount is exponential random variable with mean 1. Show

That; [6 Marks]

$$M_S(t) = p + q\frac{p}{p-t}$$

(c) If Y is a random variable with $P(x = i) = \frac{1}{4}, i = 1,2,3,4$. Find the MGF of Y[6 Marks]

4. (a). A claim amount on a certain type of insurance policy depends on a parameter α which varies from policy to policy. The mean and variance of the claim X given α are specified by;

$$E(X\backslash\alpha) = 200 + \alpha$$
$$Var((X\backslash\alpha) = 10 + 2\alpha$$

The parameter α follows a normal distribution with mean 20 and variance 4. Find the unconditional mean and variance [10Marks]

(b). Let N be the number of claims in a year. Suppose that claims $X_1, X_2, \ldots$ are independent and identically distributed random variables independent of N. Let S be the total amount claimed in one year.

Derive the Expectation of S,($E(S)$, and the variance of S,$Var(S)$ [10 Marks]