JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ACTUARIAL SCIENCE

2^{ND} YEAR 2^{ND} SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2202

COURSE TITLE: LIFE TESTING ANALYSIS

EXAM VENUE: **STREAM: ACTUARIAL**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

<u>Instructions:</u>

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1 [30marks]

a. Losses arising from a certain group of policies are assumed to follow an *Exp*(λ) distribution. You are given the following data:
   - the exact amounts $x_1, x_2, \ldots \ldots \ldots .. x_n$ paid by the insurer in respect of *n* losses
   - data from a further *m* losses, in respect of which the insurer paid an amount *M* .

   The actual loss amounts exceeded *M*, but you don't know by how much.
   Calculate the maximum likelihood estimate of λ. [4marks]
b. State the fundamental relationship between: $F_t,\ S_t,\ f_t, \mu_t$ [4marks]
c. State three scenarios when data is said to be left censored. [3marks]
d. What does the function $S_{29}(36)$ represent in survival models [2marks]
e. Butterflies of a certain species have short lives. After hatching, each butterfly experiences a lifetime defined by the following probability distribution:

Lifetime (days)	**Probability**
1	0.10
2	0.30
3	0.25
4	0.20
5	0.15

Calculate λ_j j =1,2,...,5 (to 3 decimal places) and sketch a graph of the discrete hazard function. [5marks]

f. Using ELT15 (Males) mortality, find the curtate expectation of life e_x for: [6marks]
   (i) a new-born baby
   (ii) a 21-year old actuarial student
   (iii) a 70-year old pensioner

g. If μ_x takes the constant value 0.001 between ages 25 and 35, calculate the probability that a life aged exactly 25 will survive to age 35. [3marks]
h. For a force of mortality μ_x that is known to follow Gompertz' Law, calculate the parameters *B* and *c* if $\mu_x = 0.017609$ and $\mu_{55} = 0.028359$. [3marks]

Question 2 [20marks]

A chef specializing in the manufacture of fluffy meringues uses a *Whiskmatic* disposable electric kitchen implement. The *Whiskmatic* is rather unreliable and often breaks down, so the chef is in the habit of replacing the implement in use at a given time, shortly before an important social function or after making the 1,000th fluffy meringue with that implement.
The following times until mechanical failure (no asterisk) or replacement whilst in working order (asterisk) were observed (measured in days of use):
17, 13, 15*, 7*, 21, 18*, 5, 18, 6*, 22, 19*, 15, 4, 11, 14*, 18, 10, 10, 8*, 17

a. Define $m, n, k, t, j,\ d_j$, $c_j$and n_j for these data, assuming that censoring occurs just after the failures were observed. [4marks]
b. Calculate the Kaplan-Meier estimate of $F(t)$. [16marks]

Question 3 [20marks]

A group of six lives was observed over a period of time as part of mortality investigation. Each of the lives was under observation at all ages from age 55 until they died or were censored. The table below shows the sex, age at exit and reason for exit from the investigation.

Life	**Sex**	**Age at exit**	**Reason for exit**
1	M	56	Death
2	F	62	Censored
3	F	63	Death
4	M	66	Death
5	M	67	Censored
6	M	67	Censored

The following model has been suggested for the force of mortality:

$\mu(x|Z = z) = \mu_0(x)e^{\beta z}$

where:

- x denotes age
- $\mu_0(x)$ is the baseline hazard
- Z = 0 for males and Z = 1 for females.

a. Write down the partial likelihood for these observations using the model above. [8marks]

b. Calculate the maximum likelihood. [12marks]

Question 4 [20marks]

A medical study was carried out between 1 January 2001 and 1 January 2006, to assess the survival rates of cancer patients. The patients all underwent surgery during 2001 and then attended 3-monthly check-ups throughout the study.

The following data were collected:

For those patients who died during the study exact dates of death were recorded as follows:

Patient	*Date of surgery*	*Date of death*
A	1 April 2001	1 August 2005
B	1 April 2001	1 October 2001
C	1 May 2001	1 March 2002
D	1 September 2001	1 August 2003
E	1 October 2001	1 August 2002

For those patients who survived to the end of the study:

Patient	*Date of surgery*
F	1 February 2001
G	1 March 2001
H	1 April 2001
I	1 June 2001
J	1 September 2001
K	1 September 2001
L	1 November 2001

For those patients with whom the hospital lost contact before the end of the investigation:

Patient	*Date of surgery*	*Date of last check-up*
M	1 February 2001	1 August 2003
N	1 June 2001	1 March 2002
O	1 September 2001	1 September 2005

(i) Explain whether and where each of the following types of censoring is present in this investigation:
(a) type I censoring. [2marks]
(b) interval censoring. . [2marks]
(c) informative censoring. [2marks]

(ii) Calculate the Kaplan-Meier estimate of the survival function for these patients. State any assumptions that you make. [8marks]
(iii) Hence estimate the probability that a patient will die within 4 years of surgery.[2marks]

(iv) State four applications of survival models in assurance contracts [4marks]

Question 5 [20marks]

a. i) T_x denotes the future lifetime of a life currently aged x . Write down the probability density function of T_x. [2marks]

ii) Using your answer to (i), show that: [6marks]

$\frac{\partial}{\partial s} \log {}_sP_x = \mu_{x+s}$ and

${}_tP_x = \exp\{-\int_0^t \mu_{x+s} ds\}$

iii) In a certain population, the force of mortality is given by:

$60 < x \leq 70$	0.01
$70 < x \leq 80$	0.015
$x > 80$	0.025

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83. [4marks]

a. Show that, if mortality experience conforms to Gompertz' Law, then:

$$-\log(-logP_x) = \log\left[\frac{logc}{B(c-1)}\right] - xlogc$$

Suggest how this property could be used. [8marks]