JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ACTUARIAL SCIENCE

2ND YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2206

COURSE TITLE: ACTUARIAL MATHEMATICS I

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

<u>**Instructions:**</u>

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1 [30marks]

a. Prove algebraically and by general reasoning that:

$$\ddot{a}_{\overline{n}|} = a_{\overline{n-1}|} + 1.$$

[5marks]

b. Show that the effective rate of discount is

$$d_n = \frac{A(n) - A(n-1)}{A(n)}$$

[5marks]

c. A company expects to receive for the next five years a continuous cashflow with a rate of payment of 100×0.8^t at time t years. Calculate the present value of this cashflow assuming a constant force of interest of 8% pa. [5marks]

d. A company expects to receive for the next five years a continuous cashflow of £350 pa. It also expects to have to pay out £600 at the end of the first year and £400 at the end of the third year. Calculate the net present value of these cashflows if $V(t) = 1 - {}^t/_{100}$ for $0 \leq t \leq 5$.

[5marks]

e. £4,600 is invested at time 0 and the proceeds at time 10 are £8,200. Calculate A (7, 10) if A (7, 10) if $A\,(0.9) = 1.8$, $A\,(2,4) = 1.1$, $A\,(2,7) = 1.32$, $A\,(4,9) = 1.45$ [5marks]

f. An investor is to pay £800 for a property. The investor will then be entitled to receive rent payments for 99 years payable at the end of each year at a constant rate for the first 33 years, increasing to double that rate for the next 33 years and three times that rate for the remaining 33 years. The value of the property at the end of the 99 years is expected to be £250,000. Find the amount of the rent payable in the first year, if the investor expects to obtain a rate of return of 8% on the purchase. [5marks]

Question 2 [20marks]

a. A businessman is considering an investment which requires an initial outlay of £60,000 and a further outlay of £25,000 in eight months' time. Starting two years after the initial outlay, it is estimated that income will be received continuously for four years at a rate of £5,000 per annum, increasing to £9,000 per annum for the next four years, then increasing to £13,000 per annum for the following four years and so on, increasing by £4,000 per annum every four years until the payment stream stops after income has been received for 20 years (ie 22 years after the initial outlay). At the point when the income ceases, the investment can be sold for £50,000. Calculate the net present value of the project at a rate of interest of 9% per annum effective

[12marks]

b. A company has just bought an office block for £5m, which it will rent out to a number of small businesses. The total rent for the first year will be £100,000, increasing by 4% pa compound in each future year. It will be sold after 20 years for £7.5m. Assuming that rent is paid in the middle of each year, calculate the yield the company will obtain on this investment. Ignore tax.[8marks]

Question 3 [20marks]

a. A bank account pays 10% effective annual interest rate over 5 years. Find the equivalent:]

(i)   simple annual interest rate   [2marks]
(ii)   effective monthly interest rate   [2marks]
(iii)   effective biennial interest rate   [2marks]
(iv)   effective annual discount rate   [2marks]
(v)   simple annual discount rate.   [2marks]

b. A woman has invested some money in a company run by some ex-criminals. In return for the investment she expects to receive £100 at the end of each of the next ten years. Interest rates are 5% pa effective. Calculate the present value of her investment by:

(i)   ignoring the possibility that the payments might not be made.   [2marks]
(ii)   assuming the probability of receiving the first payment is 0.95, the second payment is 0.9, the third payment is 0.85 etc.   [6marks]
(iii)   increasing the force of interest by 0.04652.   [2marks]

Question 4 [20marks]

a. Calculate the present value as at 1 January 2003 of an annuity payable annually in arrear for 15 years. The first payment is 500 and subsequent payments increase by 3% per annum compound. Assume effective rates of interest of 10% per annum.   [10marks]

b. Find the present value as at 1 June 2004 of payments of £1,000 payable on the first day of each month from July 2004 to December 2004 inclusive, assuming a rate of interest of 8% per annum convertible quarterly.   [10 marks]

Question 5 [20marks]

Prove that:

$$a_{\overline{n}|} = \frac{1-v^n}{i}$$

[5marks]

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$$

[5marks]

$$\ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d}$$

[5marks]

$$\overline{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{\delta}$$

[5marks]