



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL  
SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ACTUARIAL  
SCIENCE**

**3<sup>rd</sup> YEAR 2<sup>ND</sup> SEMESTER 2023/2024 ACADEMIC YEAR**

**MAIN CAMPUS**

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**COURSE CODE: WAB 2301**

**COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATION II**

**EXAM VENUE: STREAM: EDUCATION, ACTUARIAL**

**DATE: EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question one (compulsory) and any other two questions.**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

## **QUESTION 1 [30 MARKS]**

- a. Differentiate between mortality and morbidity [4marks]
- b. The disreputable insurance company Honest Sid's Mutual had mixed fortunes in the year 2006. At both the start and the end of the year 547 policies were in force in respect of policyholders aged 40, but these figures do not tell the whole story. There was adverse publicity early in the year linking the company's investment managers with a gambling syndicate. As a result, many policyholders "took their money elsewhere". Following a successful marketing campaign offering a free toaster to all applicants, the number of policyholders aged 40 rose from 325 at 1 June 2006 to 613 at 1 September 2006. Calculate the value of  $E_{40}^C$  approximately. [5marks]
- c. State and explain the five forms of selection [5marks]
- d. State and explain the factors which contribute to the variation in mortality and morbidity by region and according to the social and economic environment. [5marks]
- e. Define the term selection [2marks]
- f. State two advantages and disadvantages of reference to graphical method as a method of graduation [4marks]
- g. Describe how occupation affects morbidity and mortality. [3marks]
- h. "A directly standardised mortality rate is most heavily influenced by the older ages." True or false? Explain. [2marks]

## **QUESTION 2 [20 MARKS]**

- a. Let  $P(t)$  denote the population of a certain country at time  $t$  ( $t \geq 0$ ).  $P = P(t)$  is known to satisfy the equation:
- $$\frac{dP}{dt} = \rho \cdot P - kP^3 \quad (t \geq 0),$$
- where the present population,  $P(0)$ , is known and  $\rho, k$  are positive constants.
- By substituting  $\mu = P^{-2}$ , find a first-order linear differential equation for  $\mu = \mu(t)$  [3marks]
  - Solve that differential equation by the integrating factor method. [7marks]
  - Hence solve the original equation for  $P(t)$ . [3marks]
  - What is the limit of  $P(t)$  as  $t \rightarrow \infty$ . [2marks]
- b. Explain why graduated rates, rather than crude estimates of mortality rates are used in the construction of standard mortality tables. [5marks]

**QUESTION 3 [20 MARKS]**

In a given population with mortality rates has ranges between 40-44 were estimated by fitting a straight line  $a + by$  to the crude values of  $\ln(m_x/x_x)$ .

<i>Age y</i>	40	41	42	43	44
<i>Number of deaths in 2022</i>	225	325	400	410	440

The initial exposed to risk in 2003 was approximately 500,000 at each age.

- i. Obtain the equation of  $m_x$  [6marks]
- ii. Test whether this model (with estimated parameter values of  $a = -8.9446$  and  $b = 0.211634$ ) can be considered to give a good fit to the data shown in the table above for 2022. [14marks]

**QUESTION 4 [20 MARKS]**

- a. Define the following terms giving detailed formulae for each:
  - i. Crude Mortality Rate [4marks]
  - ii. Directly Standardised Mortality Rate [4marks]
  - iii. Indirectly Standardised Mortality Rate [4marks]
- b. You are given the following statistics in respect of the population of Urbania:

	<i>Males</i>		<i>Females</i>	
<i>Age band</i>	<i>Exposed to risk</i>	<i>Observed Mortality rate</i>	<i>Exposed to risk</i>	<i>Observed Mortality rate</i>
20–29	125,000	0.00356	100,000	0.00125
30–39	200,000	0.00689	250,000	0.00265
40–49	100,000	0.00989	200,000	0.00465
50–59	90,000	0.01233	150,000	0.00685

Calculate the directly and indirectly standardised mortality rates for the female lives, using the combined population as the standard population. [8marks]

### **QUESTION 5 [20 MARKS]**

The mortality experience of some whole of life assurance policyholders has been compared with a standard mortality table for assured lives. The following is an extract from the data:

Age, $x$	Actual deaths, $\theta_x$	Expected deaths, $E_x q_x^s$	$\theta_x - E_x q_x^s$
60	37	42.88	-5.88
61	40	61.73	-21.73
62	28	38.06	-10.06
63	41	47.23	-6.23
64	34	40.36	-6.36
65	40	49.98	-9.98
66	27	25.13	1.87
67	15	22.25	-7.25
68	16	26.23	-10.23
69	30	27.61	2.39
70	23	25.11	-2.11
Total	331	406.56	-75.57

- a. Carry out a comparison between the actual and expected mortality experience, using the following statistical tests:
- Chi-squared test [5marks]
  - Cumulative deviations test [5marks]
  - Serial correlations test. [5marks]

You should state the appropriate null hypothesis and, for each test, the conclusion reached with regard to this hypothesis

- b. Summarise what you can infer about the mortality experience of these policyholders from your analysis, giving your reasons. [5marks]