JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR BACHELOR OF ACTUARIAL SCIENCE 2023/24

MAIN REGULAR

COURSE CODE: WAB 2402

COURSE TITLE: Risk Mathematics

EXAM VENUE **STREAM: B.Sc. Actuarial Science Year Four**

DATE:…………… **EXAM SESSION: ONE**

TIME: 2 HOURS

<u>Instructions to the Candidate:</u>

1. **Answer ALL in-Section A and any other two questions only in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

SECTION A

QUESTION ONE: [30 Marks]

a. Find the mean and variance of a random variable X with the Moment Generating Function (MGF) given by; [6 Marks]

$$M_X(t) = (1 - \tfrac{t}{5})^{-1}$$

b. Suppose that a market portfolio M is efficient, prove that an expected return $\bar{r_i}$ of any asset i satisfies the relation:

$$\bar{r_i} = r_f + \beta_i(\bar{r}_m - r_f)$$

Where r_f is risk free rate, β_i is beta of security i and $\bar{r}_m$ is expected market return'

[6 Marks]

c. A claim amount on a certain type of insurance policy depends on a parameter α which varies from policy to policy. The mean and variance of the claim X given α are specified by;

$$E(X\backslash\alpha) = 200 + \alpha$$
$$Var((X\backslash\alpha) = 10 + 2\alpha$$

The parameter α follows a normal distribution with mean 20 and variance 4. Find the unconditional mean and variance [6 Marks]

d. Let N be the number of claims in a year. Suppose that claims $X_1, X_2, \ldots$ are independent and identically distributed random variables independent of N. Let S be the total amount claimed in one year.
Derive the Expectation of S,($E(S)$, and the variance of S,$Var(S)$ [7 Marks]

e. Investor A has an initial capital of $Ksh.\,100$ and utility function of the form $U(w) = log(w)$ where w is the wealth at any time. Investment Z offers a return of -18% or $+24\%$ with equal probabilities.
   i. What is the expected utility if she invests nothing in Z? [2 Marks]
   ii. What is her expected utility if she invests entire in Z? [3 Marks]

SECTION B: [40 Marks]

QUESTION TWO: [20 MARKS]

1. The random variable Y follows a Poisson distribution with parameter θ.

   i. Derive the probability mass function (p.m.f) of Y [6 Marks]

   ii. Using the probability mass function in (i) above, derive the Moment Generating Function (MGF) of Y [6 Marks]

   iii. Derive the mean and variance of Y using the derived Moment Generating Function. [8 Marks]

QUESTION THREE: [20 MARKS]

(a) A Poisson claim process has a security loading $\theta = \frac{2}{5}$ and claim size density function;

$f(x) = \frac{3}{2}e^{-3x} + \frac{7}{2}e^{-7x}, x > 0$

i. Derive the Moment Generating Function for the claim size and state the values of t for which it is valid. [6 Marks]

ii. Calculate the value of the Adjustment Coefficient [6 Marks]

(b) A random variable X follows a gamma distribution with parameter α and θ.

Derive the Moment generating function of X. [6 Marks]

(c) State two features of generalized extreme value (GEV) distributions.

[2 Mark]

QUESTION FOUR: [20 MARKS]

(a) Given the initial capital u, derive the integral equation of ruin probability given a premium inflow C at time t with a stochastic claim payout of amount X.

[10 Marks]

(b) Define: [4 Marks]

i. Adjustment Coefficient
ii. Safety Loading

(c) Derive the rate of premium inflow [6 Marks]

QUESTION FIVE: [20 MARKS]

(a) An insurance company is earning Ksh.13200 per day. It receives in average 20 claims per day with average claim size of Ksh.600. Find the relative safety loading. Assuming that the claims sizes are exponentially distributed, calculate the ruin probability of the company at the time moment when its wealth is equal to Ksh.25 000. Find the wealth of the company such that the probability of possible ruin in the future is less than 0.01%. [6 Marks]

(b) Show that the (percent) return of the portfolio satisfies;

$r_p = w_A r_A + w_B r_B,$

i.e. the portfolio return is equal to the weighted average of the returns of its composite assets. [6 Marks]

(c) Claim on a portfolio insurance follow a poisson distribution with parameter α. The insurance company calculates the premium using premium loading factor θ and has the initial Surplus of U.

i. Define the surplus process $U(t)$ [2 Maks]

ii. Define the probabilities $\varphi(U,t)$ and $\varphi(U)$ [3 Marks]

iii. Explain how $\varphi(U,t)$ and $\varphi(U)$ depend on α [3 Marsk]