JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ACTUARIAL SCIENCE

4TH YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2404

COURSE TITLE: COMPUTATIONAL FINANCE

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

<u>**Instructions:**</u>

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1 [30marks]

a. State the advantages of the Hull & White model over the single factor Vasicek model. [5marks]

b. Define the following terminologies [5marks]
      i. Computational finance
      ii. A Martingale
      iii. Binomial tree
      iv. American call Option
      v. No arbitrage

c. Compute $E[W^4(t)]$ [5marks]

d. The stock price 6 months from expiry of an option is £42, the exercise is £40, the risk-free interest rate is 10% p.a and the volatility is 20% p.a. calculate the price of a European option written in this stock [4marks]

e. Explain the limitations of using a model with only one factor, taking into account both theoretical and empirical considerations. [5marks]

f. State five desirable characteristics of a term structure model [5marks]

g. State and explain the three main types of models used to describe interest rates mathematically [6marks]

Question 2 [20marks]

a. Let $X_t = e^{\lambda B_t - 1/2 \lambda^2 t}$, where λ is any constant. Show that X_t is a martingale with respect to F_t , the filtration associated with B_t. [15marks]

b. Show that $B_t^2 - 1$ is a martingale with respect to F_t , the filtration associated with B_t. [5marks]

Question 3 [20marks]

a. A fund must make payments of £50,000 at the end of the sixth and eighth years. Show that, if interest rates are currently 7% pa at all durations, immunization to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5- year zero-coupon bond and a 10-year zero-coupon bond. [10marks]

b. Compute $dr(t) = [k(\theta(t) - r(t)]dt + \sigma dw(t)$ [10marks]

Question 4 [20marks]

a. State Redington's conditions for immunization [3marks]

b. Explain the similarities and differences in the following three interest rate models:
      i. the Hull & White model
      ii. the Cox-Ingersoll-Ross model
      iii. the Vasicek model [9marks]

c. Derive the Black- Scholes equation [8marks]

Question 5 [20marks]

The market price of a security can be modelled by assuming that it will either increase by 12% or decrease by 15% each month, independently of the price movement in other months. No dividends are payable during the next two months. The continuously-compounded monthly risk-free rate of interest is 1%. The current market price of the security is 127.

(i)   Use the binomial model to calculate the value of a two-month European put option on the security with a strike price of 125.   [10marks]

(ii)   (ii) Calculate the value of a two-month American put option on the same security with the same strike price.   [6marks]

(iii)   (iii) Calculate the value of a two-month American call option on the same security with the same strike price.   [4marks]