JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL
4th YEAR 2nd SEMESTER 2023/2024
REGULAR (MAIN)

COURSE CODE: WAB 2406

COURSE TITLE: RISK AND CREDIBILITY THEORY.

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

a. If reported claims follow a Poisson process with rate 5 per day (and the insurer has a 24hour hotline), calculate:

i.) the probability that there will be fewer than 2 claims reported on a given day

(3 marks)

ii.) the probability that another claim will be reported during the next hour.

(3 marks)

b.

i.) State the five conditions that must hold for a risk to be insurable. (5 marks)

ii.) List five other risk criteria that would be considered desirable by a general insurer. (5marks)

c. The probability of a claim arising on any given policy in a portfolio of 1,000 one-year term assurance policies is 0.004. Claim amounts have a *Gamma* (5, 0.002) distribution. Calculate the mean and variance of the aggregate claim amount.

(6 marks)

d. A specialist insurer that provides insurance against breakdown of photocopying equipment calculates its premiums using a credibility formula. Based on the company's recent experience of all models of copiers, the premium for this year should be £100 per machine. The company's experience for a new model of copier, which is considered to be more reliable, indicates that the premium should be £60 per machine.
Given that the credibility factor is 0.75, calculate the premium that should be charged for insuring the new model.

(5 marks)

e. If X has a Pareto distribution with parameters $\lambda = 400$ and $\alpha = 3$ and N has a *Poisson* (50) distribution, calculate the expected value of S.

(4 marks)

f. A loss amount random variable has MGF:

$M(t)=0.4(1-20t)^{-2} + 0.6(1-30t)^{-3}$

Calculate the expected loss amount.

(4 marks)

QUESTION TWO

a. Claims occur according to a compound Poisson process at a rate of 0.2 claims per year. Individual claim amounts, *x*, have probability function:

$$P(x=50) = 0.7$$
$$P(x=100) = 0.3$$

The insurer's surplus at time 0 is 75 and the insurer charges a premium of 120% of the expected annual aggregate claim amount at the beginning of each year. The insurer's surplus at time *t* is denoted u(t). Find:

$P[u(2) < 0]$

(10 marks)

b. A group of policies can give rise to at most two claims in a year. The probability function for the number of claims is as follows:

Number of claims, n	0	1	2
P(N=n)	0.6	0.3	0.1

Each claim is either for an amount of 1 or an amount of 2, with equal probability. Claim amounts are independent of one another and are independent of the number of claims.

Determine the distribution function of the aggregate annual claim amount, S.

(10 marks)

QUESTION THREE

a. The distribution of the number of claims from a motor portfolio is negative binomial with parameters $k = 4{,}000$ and $p = 0.9$. The claim size distribution is Pareto with parameters $\alpha = 5$ and $\lambda = 1{,}200$. Calculate the mean and standard deviation of the aggregate claim distribution.

(10 marks)

b. The annual aggregate claims from a risk have a compound Poisson distribution with parameter 250. Individual claim amounts have a Pareto distribution with parameters $\alpha = 4$ and $\lambda = 900$. The insurer effects proportional reinsurance with a retained proportion of 75%.
Calculate the variances of the total amounts paid by the insurer and by the reinsurer.

(10 marks)

QUESTION FOUR

The figures given in the table below are the aggregate claims (in £000s) for each of four risks over a period of four years.

	Year 1	Year 2	Year 3	Year 4
Risk 1	1,892	1,975	2,309	2,278
Risk 2	2,356	2,876	3,002	3,378
Risk 3	2,890	2,489	2,424	2,551
Risk 4	1,662	1,408	1,697	2,034

Assuming that the data satisfy the assumptions of EBCT Model 1, estimate the aggregate claim amount for Risk 1 in Year 5.

(20 marks)

QUESTION FIVE

a. The claims arising during each year from a particular type of annual insurance policy are assumed to follow a normal distribution with mean $0.7P$ and standard deviation $2.0P$, where P is the annual premium. Claims are assumed to arise independently. Insurers assess their solvency position at the end of each year.
A small insurer with an initial surplus of £0.1m expects to sell 100 policies at the beginning of

the coming year in respect of identical risks for an annual premium of £5,000. The insurer incurs expenses of $0.2P$ at the time of writing each policy. Calculate the probability that the insurer will prove to be insolvent at the end of the coming year. Ignore interest.

(10 marks)

b. If the insurer expects to sell 200 policies during the second year for the same premium and expects to incur expenses at the same rate, calculate the probability that the insurer will prove to be insolvent at the end of the second year.

(10 marks)