JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2023/2024 EXAMINATIONS

MAIN REGULAR

COURSE CODE: SPB 9327
COURSE TITLE: QUANTUM MECHANICS I
EXAM VENUE: **STREAM: EDUCATION**
DATE: **EXAM SESSION:**
TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants
$\hbar = 1.054 \times 10^{-34}$ Js
Mass of proton = 1.67×10^{-27} kg
1 eV = 1.6×10^{-19} J
Mass of electron = 9.11×10^{-31} kg
Planck's constant = 6.626×10^{-34} Js

SECTION A (Compulsory)
Question 1 (30 marks)

(a) State any two postulates of quantum mechanics. (2 marks)
(b) Find the normalization constant A in the wave function $\Psi(x) = Asinkx$within the interval $0 \leq x \leq a$. (3 marks)
(c) An electron has a kinetic energy of 1 eV. Calculate its de Broglie wavelength. (3 marks)
(d) Show that the function $\Psi(x,t) = Ae^{i(kx-\omega t)} - Ae^{-i(kx+\omega t)}$is a solution to the Schroedinger equation $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$and give the conditions for this to be so. (4 marks)
(e) A wave packet is to be used for describing a particle that is localized within a region of space. Describe the composition of such a wave packet and explain how it may be realized practically. (3 marks)
(f) An electron moves in the x-direction with a speed of 3.6 x 10^6 m/s. Its speed can be measured to a precision of 1%. Calculate the precision with which its x-coordinate can be simultaneously measured. (4 marks)
(g) The phase velocity of water waves in deep water is given by

$$v = \sqrt{\frac{g}{2\pi\lambda}}.$$

Find the group velocity of a wave packet of such waves. (3 marks)
(h) Even when the rate of flow of photons in a double slit experiment is reduced to one incident photon at a time, an interference pattern still forms on the screen eventually. Explain this observation using quantum mechanics. (3 marks)
(i) Explain what is meant by a bound state in quantum mechanics. (2 marks)
(j) An electron in an infinite square well moves from level n = 4 to the level n = 1, emitting a photon of frequency 3.54 x 10^{14} Hz. Calculate the width of the well. (3 marks)

SECTION B (Answer any TWO questions in this section)

Question 2 (20 marks)

(a) A particle of mass m is in a one-dimensional potential

$$V(x) = \begin{cases} \infty, \text{if} -\infty < x < 0 \\ -V_0, \text{if} 0 < x < a \\ 0, \text{if} a < x < \infty \end{cases}.$$

i. Sketch a graph of the potential against position, and explain why a bound state can only exist in a specified region of the graph. (3 marks)
ii. For a bound state, show that a is defined by $a = \frac{1}{k_0}cot^{-1}\left(-\frac{\alpha}{k_0}\right)$where $\alpha$and k_0 are parameters. (8 marks)

(b) A particle is described by the wave function $\Psi(x,t) = e^{i(kx-\omega t)}$.
(i) Apply the momentum operator and also the energy operator $-i\hbar(d/dt)$on the function hence determine the eigenvalues in each case. (6 marks)
(ii) State, with a reason, whether or not the function represents a bound state. (3 marks)

Question 3 (20 marks)

(a) Suppose an electron has a wave function $\psi(x) = cx^3 e^{-\alpha x}$ where α is a constant.

(i) Find the constant c that ensures the given wave function is properly normalized. You may use the standard integral

$$\int x^n\, e^{-bx} dx = \frac{n!}{b^{n+1}}. \qquad \text{(6 marks)}$$

(ii) Find the expectation value of x. (5 marks)

(b) The wave functions of the two waves emerging from the two slits in a double-slit experiment are given by $\Psi_1 = A_1 e^{+i(kR_1 - \omega t)}$ and $\Psi_2 = A_2 e^{+i(kR_2 - \omega t)}$ at the point where constructive interference occurs on the screen.

i. Explain what A_1, A_2 as well as R_1 and R_2 represent. (2 marks)

ii. Obtain, in the simplest form, the probability density function of the position of a quantum particle on the screen, $|\Psi|^2$.You may use the trigonometric identities

$$cos\theta = \left(e^{+i\theta} + e^{-i\theta}\right)/2\,; \qquad cos\theta = 2cos^2\left(\frac{\theta}{2}\right) - 1 \qquad \text{(7 marks)}$$

Question 4 (20 marks)

(a) A particle is confined to an infinite square well of width a.

i. Show that the possible values of energy are given by $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$. (6 marks)

ii. Find an expression for the possible wave functions inside the well, $\psi_n(x)$. (4 marks)

(b) Use your results in (a) above to determine the expectation values of position x and of the momentum p_x. (10 marks)

Question 5 (20 marks)

(a) A solution to the one-dimensional simple harmonic oscillator for a mass m is given as $\psi(x) = Ae^{-ax^2}$.Obtain the value of a and the energy E in terms of k, m and $\hbar$ where k has the usual meaning.

(9 marks)

(b) A particle of mass m, confined to a harmonic oscillator potential $V = mx^2\,\omega^2/2$, is in a state described by the wave function

$$\Psi(x,t) = Ae^{\left(\frac{-mx^2\omega}{2\hbar} - i\frac{\omega t}{2}\right)}$$

Verify that this is a solution of the Schrödinger equation. (5 marks)

(c) Find the eigenvalues and eigenfunctions of the operator d/dx. (6 marks)